

# Scheduling with Outdated CSI: Effective Service Capacities of Optimistic vs. Pessimistic Policies

James Gross

Mobile Network Performance Group,  
UMIC Research Centre, RWTH Aachen University, Germany  
gross@umic.rwth-aachen.de

**Abstract**—The concept of the effective service capacity is an analytical framework for evaluating QoS-constrained queuing performance of communication systems. Recently, it has been applied to the analysis of different wireless systems like point-to-point systems or multi-user systems. In contrast to previous work, we consider in this work slot-based systems where a scheduler determines a packet size to be transmitted at the beginning of the slot. For this, the scheduler can utilize outdated channel state information. Based on a threshold error model, we derive the effective service capacity for different scheduling strategies that the scheduler might apply. We show that even slightly outdated channel state information leads to a significant loss in capacity in comparison to an ideal system with perfect channel state information available at the transmitter. This loss depends on the ‘risk-level’ the scheduler is willing to take which is represented by an SNR margin. We show that for any QoS target and average link state there exists an optimal SNR margin improving the maximum sustainable rate. Typically, this SNR margin is around 3 dB but is sensible to the QoS target and average link quality. Finally, we can also show that adapting to the instantaneous channel state only pays off if the correlation between the channel estimate and the channel state is relatively high (with a coefficient above 0.9).

## I. INTRODUCTION

At the heart of wireless network analysis is the modeling of the wireless channel and its interaction with the network stack. Due to the complexity of this interaction, precise theoretical understanding of wireless network performance is still an ongoing research issue [1]. While straightforward metrics like throughput or bit error rate are usually the performance metrics of choice at the physical layer, higher layer performance analysis requires queuing-related metrics. However, applying traditional queuing theory analysis to service processes resulting from representative wireless channel models provides only the characterization of average system performance. In contrast, from an application point of view, we are more interested in distributions especially regarding the delay.

To date, at least two related approaches exist that address this problem. The first one is based on stochastic network calculus. Recently, there is interest in formulating (stochastic) service curve models for wireless systems [2], [3]. While being

(stochastic) worst-case service characterizations, this approach yields (probabilistic) delay and backlog bounds for arbitrarily-shaped arrival processes. A further strength of this approach is its applicability to multi-hop transmission scenarios [4]. On the other hand, based on the initial analysis by Wu et al. [5] a second (related) line of research focuses on determining the so called effective service capacity of wireless systems [6], [7]. In contrast to stochastic service curves, this approach targets directly at approximating the steady-state queuing behavior of a constant-rate source that is served by a wireless system. More precisely, by analyzing the effective service capacity of a link one yields the maximum arrival rate that this link can sustain under given quality-of-service constraints at the link layer (either delay or buffer occupancy along with a violation probability for the delay or buffer occupancy). Hence, the effective service capacity is a metric that can be used directly for admission control or network dimensioning.

In this work, we contribute to the modeling of wireless system performance based on the effective service capacity approach. The focus of our work is on the impact of outdated channel state information available at the transmitter. Typically, derivations of the effective service capacity are based on perfect channel state information at the transmitter. In addition, it is often assumed that a transmission system can adapt on a per-symbol base to the changes of the wireless channel. In contrast, in this paper we develop analytical expressions under more realistic assumptions. First, we account for the fact that systems are typically slotted and the transmitter has to decide prior to the upcoming slot for a certain data amount to be transmitted. We refer to this decision process in the following as *scheduling*. Second, we account for link layer packet losses. This is closely related to the time-slotted design of wireless systems as the transmitter most likely does not know the channel state perfectly during the upcoming slot. As a consequence, packets might be erroneous if the transmitter overestimates the channel state during the upcoming slot. Third, based on a loss model, the scheduling approach at the transmitter in combination with the quality of available channel state information becomes crucial for the resulting performance. Our major contribution is to characterize the effective service capacity for different scheduling strategies (optimistic vs. pessimistic) depending on the quality of channel state information in a time-correlated fading channel. We can show that scheduling on outdated CSI only pays off if a

This work was funded by the DFG Cluster of Excellence on Ultra High-Speed Mobile Information and Communication (UMIC), German Research Foundation grant DFG EXC 89.

significant *SNR margin* is taken into account. For any given average channel quality and QoS target there exists an optimal SNR margin which should be used for scheduling data.

The paper is structured as follows: In Section II we present the system model, give a precise problem statement, and summarize the effective service capacity approach. Then, in Sections III, IV and V we present the analytical work describing the effective service capacity for a transmitter with average CSI, perfect CSI and outdated CSI under different scheduling strategies. In Section VI we validate our analysis and discuss some insights from numerical evaluation. Finally, in Section VII we conclude the paper.

## II. PRELIMINARIES

In the following we first present the system model and then the problem statement. Section II-C gives a brief overview of the effective service capacity framework, while Section II-D summarizes related work.

### A. System Model

We consider a simple scenario with one transmitter/receiver pair. Time is divided into frames of length  $T_f$ . A constant data flow originates at the transmitter and needs to be transmitted to the receiver. The flow arrival rate equals  $r$  bits per frame duration  $T_f$ . The transmission of this data is subject to quality-of-service requirements  $\{d, \mathcal{P}_d\}$  where  $d$  stands for a maximum tolerable delay and  $\mathcal{P}_d$  denotes the maximum outage probability, i.e. the probability that the delay target is not met. Data that can not be transmitted immediately at the transmitter is put into a FIFO queue (of infinite size). Finally, we denote the cumulative arrival process to the link layer at the transmitter up to frame  $i$  by  $A_i = \sum_1^i a_i = i \cdot r$ .

Per frame, the transmitting station has one slot of duration  $T_s$  to forward data to the receiver. These slots are spaced regularly within the frames such that two consecutive slots are separated in time by a duration of  $T_f - T_s$  (during which other stations are actively transmitting data but are not considered any further in this paper). Per slot the station transmits  $N$  symbols. These symbols are conveyed with a power of  $P_{tx}$ . During each slot  $i^1$ , the transmitter takes  $\sigma_i$  bits out of the queue (if it is backlogged) and transmits the data as a packet over the wireless channel. Denote the cumulative service process up to (and including) slot  $i$  by  $S_i = \sum_1^i s_i$  where  $s_i$  denotes the amount of successfully transmitted bits during period/slot  $i$ . Depending on the channel conditions and the channel state information available at the transmitter, the packet transmission per slot either fails or succeeds as explained in the following.

The wireless channel between transmitter and receiver is affected by a randomly changing channel gain. This channel gain is composed of two factors. Transmitter and receiver are separated by a certain fixed distance which results in a constant

path loss  $h_{PL}^2$ . In addition, due to moving objects in the multi-path environment, the received signal is also attenuated by time-varying, random fading. Denote by  $h_i^2$  the random fading channel gain during frame  $i$ . We assume a correlated, slowly-fading Rayleigh-distributed process with Jakes power spectrum density. Therefore the fading is constant during the duration of a transmission slot. Although the fading process is assumed to be correlated, consecutive transmission slots are assumed to have independent fading gains due to the large time duration in between. Thus, the resulting (random) signal-to-noise ratio(SNR) per slot  $i$  is given by:

$$\gamma_i = \frac{P_{tx} \cdot h_{PL}^2 \cdot h_i^2}{n^2}, \quad (1)$$

where  $n^2$  denotes the power of the noise process.

Depending on the SNR per slot, a different amount of information can be conveyed from the transmitter to the receiver. Prior to slot  $i$  the transmitter schedules a potentially varying amount of  $\sigma_i$  bits for transmission. Hence, each symbol of the frame represents then  $\sigma_i/N$  bits. This scheduling decision most likely depends on the accuracy of the channel state information and will be discussed in more detailed in later sections. Nevertheless, the wireless channel has a certain transport capacity during slot  $i$  which is assumed to be:

$$c_i = N \cdot \log_2(1 + \beta \cdot \gamma_i), \quad (2)$$

where  $0 < \beta < 1$  is a scaling factor. If the transport capacity  $c_i$  is bigger or equal than the size of the scheduled data packet  $\sigma_i$ , the transmission is received successfully. Hence, we have  $s_i = \sigma_i$ . Otherwise, the packet is lost completely, i.e. if  $c_i < \sigma_i$  then  $s_i = 0$ . Thus, we assume a threshold-based error model for our investigations, which is accurate for larger packet sizes [8]. A lost packet is indicated by a missing acknowledgment. Then, the data of the transmitted packet remains in the queue until the next transmit slot comes up.

### B. Problem Statement

In this paper we are interested in the impact of different degrees of channel knowledge on the quality-of-service (QoS) provisioning of the considered data flow. Recall that the data flow is subject to a delay target  $d$  and a maximum allowed outage probability  $\mathcal{P}_d$ . The question arises if the system can meet these QoS targets facing the random fluctuations of the wireless channel gains. This depends essentially on the accuracy of channel state information at the transmitter and the corresponding scheduling decisions per transmission slot. In particular we consider the following three cases :

- **Average SNR:** The transmitter does not know the instantaneous channel state information but tracks the average SNR. Therefore, it schedules during for each slot a packet with a fixed size depending on the average SNR. As the transmitter is unaware of the upcoming channel state, packets will be lost from time to time due to fading and the resulting insufficient channel capacity.
- **Perfect Instant. SNR:** The transmitter has perfect, instantaneous channel state information prior to the upcoming transmission slot. Hence, the transmitter schedules

<sup>1</sup>As each frame contains exactly one transmission slot, we use the index  $i$  in the following interchangeably to denote the frame or the corresponding transmission slot of the transmitter of interest per frame.

packets of varying size per slot. Obviously, no packet will be lost at the receiver.

- **Outdated Instant. SNR:** The transmitter has outdated instantaneous channel state information and is aware of the imperfect channel state information. In this case the transmitter again schedules packets of varying size. However, as the channel state information is not perfect, packets will be lost from time to time. This depends essentially on two factors: The statistical deviation between the channel state information at the transmitter and the real channel state during the upcoming transmission slot as well as the scheduling strategy at the transmitter. Assuming the transmitter to be aware of the erroneous channel state information, it can schedule packets more carefully to prevent packet losses. Hence, scheduling strategies can be either optimistic or pessimistic.

In all three cases, we are interested in obtaining analytical expressions for the maximum arrival rate for which the given QoS targets can be met. This analysis is based on effective service capacity framework. In contrast to related work regarding the effective service capacity of wireless systems, the novelty of our work lies in the consideration of:

- 1) Packet losses at the receiver due to imperfect channel state information at the transmitter;
- 2) Scheduling policies that take imperfect channel state information into account in order to increase the reliability.

Notice that the effective service capacity of a system with perfect channel state information has already been derived in [7]. In the following, we give a brief overview of the effective service capacity framework.

### C. Effective Service Capacity

The mathematical framework of the effective service capacity allows to approximate the distribution of the steady-state queue length of a stable queuing system. It is therefore a tool for analysis of arbitrary service processes in a queuing system. The framework was originally applied to characterize the queue length for arbitrary arrival processes (source flows) which are served by a constant rate queuing system. In this context, deriving the so called *effective bandwidth* of the arrival process allows to bound the queue length distribution [9]. Interestingly, this analysis technique can also be turned 'up side down' such that the effective service capacity of a random service process has to be derived in order to bound the queue length distribution assuming constant arrivals. In the following we give a brief introduction to this analytical framework. The starting point for the analysis is Reich's equation which states that for the considered queuing system the queue length at time  $i$  is given by:

$$Q_i = \max_{0 \leq k \leq i} ((i - k) \cdot r - (S_i - S_k)) . \quad (3)$$

Let us consider that the arrival and service process are stationary. Furthermore, assume that the queue is stable as the average service rate is larger than the average arrival rate. Hence, the random queue length  $Q_i$  at time  $i$  converges to

the steady-state random queue length  $Q$ . We are interested in characterizing the long-term statistics  $\text{Pr.}\{Q\}$  of the queue length. The framework of effective service capacity gives us the following upper bound:

$$\text{Pr.}\{Q > x\} \leq K \cdot e^{-\theta^* \cdot x} , \quad (4)$$

where  $K$  is the probability that the queue is non-empty and  $\theta^*$  is the so called quality-of-service exponent. Due to several mathematical derivation steps [10], for a constant bit rate source with  $r$  bits per time unit arrival rate, the exponent  $\theta^*$  has to fulfill the following constraint:

$$r < \frac{\Lambda(-\theta^*)}{\theta^*} . \quad (5)$$

$\Lambda(\theta)$  is called the log-moment generating function of the increments of the cumulative service process  $S_i$  defined as (assuming the increments to be stationary as well):

$$\Lambda(\theta) = \lim_{i \rightarrow \infty} \frac{1}{i} \log \text{E} \left[ e^{\theta \cdot (S_i - S_0)} \right] . \quad (6)$$

Finally, the ratio  $\Lambda(-\theta)/\theta$  is called the effective service capacity, as the exponential decay of the distribution in Equation (4) is only witnessed if the ratio  $\Lambda(-\theta)/\theta$  is bigger than the constant arrival rate  $r$  of the source for some  $\theta^*$ .

So far we have only considered the random queue length. Denote by  $D_i$  the random queuing delay of the head-of-line bit during frame  $i$ . This random variable converges in the long-run to the random steady state queuing delay  $D$  of the head-of-line bit. As the arrival process has a fixed rate, the steady-state queue length statistics are related to the steady-state delay statistics of the head-of-line bit. Hence, a queue length of  $Q = q$  is associated with a current delay of the head-of-line bit of  $D = q/r$ . This yields the following approximation for the steady-state delay distribution which is based on Equation 4:

$$\text{Pr.}\{D > d\} \leq K \cdot e^{-\theta^* \cdot r \cdot d} . \quad (7)$$

A considerable challenge in determining the effective service capacity is the characterization of the log-moment generating function. If the service process  $s_i$  can be assumed to be i.i.d., a convenient simplification is to obtain the log-moment generating function via the law of the large numbers [7]. Hence, the effective service capacity can be obtained by:

$$\frac{\Lambda(-\theta)}{\theta} = \lim_{i \rightarrow \infty} \frac{1}{i \cdot \theta} \log \text{E} [e^{-\theta \cdot s_i}] = \text{E} [s_i] - \frac{\theta}{2} \text{Var} [s_i] . \quad (8)$$

It is therefore sufficient to determine the average and the variance of the instantaneous service process  $s_i$ .

The above analysis allows to determine a bound on the maximum outage probability if the (constant) arrival rate is given. In contrast, we can also fix the delay and outage target and derive the maximum arrival rate  $r^*$  that can be supported by the random service process. From Equation (7) we obtain the following (upper bounding  $K$  by 1):

$$\frac{-\ln(\mathcal{P}_d) + \ln(K)}{d} \geq r \cdot \theta \Leftrightarrow r^* \cdot \theta \approx -\frac{\ln(\mathcal{P}_d)}{d} , \quad (9)$$

where the approximation results from the fact that upper bounding  $K$  by one can underestimate the maximum sustainable rate if especially the delay target is quite low. Next, from Equation (5) and (8) we obtain in general for  $\theta$ :

$$\theta = 2 \cdot \frac{\mathbb{E}[s_i] - r}{\text{Var}[s_i]}. \quad (10)$$

We use this expression and substitute it in Equation (9). We then obtain the following relationship for the maximum sustainable rate  $r^*$  which has been first proposed by Soret et al. [7]:

$$\begin{aligned} 2 \cdot \mathbb{E}[s_i] \cdot r^* - (r^*)^2 &\approx -\frac{\ln(\mathcal{P}_d)}{d} \cdot \text{Var}[s_i] \Rightarrow \\ r^* &\approx 0.5 \cdot \mathbb{E}[s_i] + \\ &0.5 \cdot \sqrt{(\mathbb{E}[s_i])^2 + \frac{2 \cdot \ln(\mathcal{P}_d)}{d} \cdot \text{Var}[s_i]}. \end{aligned} \quad (11)$$

In the following, we use this equation to determine the maximum sustainable source rate. The major difficulty that we will face is obtaining the mean and variance of the instantaneous service process depending on the scheduling strategy of the transmitter.

#### D. Related Work

The notion of the effective service capacity for wireless communications has been introduced by Wu et al. in [5]. This initial work considered a single wireless fading channel with perfect channel state information at the transmitter and the possibility to adapt to the channel state on a per-symbol base. Difficulties in characterizing the log-moment generating function for a correlated wireless channel forced the authors to consider special cases like low-SNR-regime etc. The work was extended afterwards by numerous further contributions of Wu et al. considering for example point-to-point communication over frequency-selective fading channels [11], multi-user communication over a single flat-fading channel [12], as well as multi-user communications over parallel, down-link fading channels [13]. All these works did not take the impact of outdated channel state information into account. Further contributions extending the effective service capacity to other wireless system scenario have been presented in [14] (extension to point-to-point communication via adaptive resource allocation in multi-carrier systems) or [7] (extension to point-to-point communication of variable rate sources over a single, correlated fading channel).

Closest to our work are two recent contributions. In [15] the authors study the effective capacity over wireless channels with fixed-rate transmission where the transmitter has no channel state information. This work incorporates the impact of losses but focuses on energy-efficiency instead of considering the relationship between outdated channel state information and associated scheduling strategies at the transmitter. In contrast, Femenias et al. consider in [16] the effective service capacity of adaptive modulation and coding over a correlated Rayleigh-fading wireless channel. The authors model the service process by a Markov chain and consider packet losses. Nevertheless, the work focuses on perfect channel state

information at the transmitter. Again, scheduling strategies for maximizing the effective service capacity in case of outdated channel state information is not taken into account.

### III. EFFECTIVE SERVICE CAPACITY WITH AVERAGE SNR

In this section we consider the effective service capacity where the transmitter is only informed about the average channel state, i.e. the transmitter knows the average SNR of the link given by  $\bar{\gamma} = \bar{P}_{\text{tx}} \cdot h_{\text{PL}}^2 / n^2$ . Recall that we assume a constant fading gain per slot. More specifically, we model the channel as a Rayleigh-fading wireless channel which implies that the random SNR  $\gamma_i = \bar{\gamma} \cdot h_i^2$  during slot  $i$  is exponentially distributed. As the transmitter knows only the average channel state, we restrict our analysis to the case that the transmitter always schedules a fixed amount of data  $\sigma_i = \lambda \cdot N$  per slot. Hence, per symbol the system conveys  $\lambda$  bits.

In this case, the service process  $s_i$  takes values of either 0 or  $\lambda \cdot N$  depending on the outage probability of the wireless channel. As we are dealing with a constant packet of size  $\lambda \cdot N$  bits, at least an SNR of  $\Gamma_\lambda = (2^\lambda - 1) / \beta$  is needed to receive the packet successfully. This allows us to compute the probability of the service process  $s_i$  to take the value  $\lambda \cdot N$  which is given by:

$$\Pr. (s_i = \lambda \cdot N) = \int_{\Gamma_\lambda}^{\infty} \frac{1}{\bar{\gamma}} e^{-\frac{x}{\bar{\gamma}}} dx = e^{-\frac{2^\lambda + 1}{\bar{\gamma} \cdot \beta}} = p_\lambda. \quad (12)$$

Likewise, the probability for the service rate to be 0 is given by  $1 - p_\lambda$ . Hence, the resulting random variable  $s_i$  is a scaled Bernoulli random variable.

From the Bernoulli characteristic of the service process at slot  $i$  and our general approach to characterize the effective service capacity given by Equation (8), we therefore obtain for the mean and variance:

$$\mathbb{E}[s_i] = m_s = p_\lambda \cdot \lambda \cdot N, \quad (13)$$

$$\text{Var}[s_i] = p_\lambda \cdot \lambda^2 \cdot N^2 \cdot (1 - p_\lambda) = m_s \cdot (\lambda \cdot N - m_s). \quad (14)$$

Substituting these values in Equation 8, we obtain the effective service capacity<sup>2</sup> depending on the scheduling choice  $\lambda$ . Based on this expression for the effective service capacity, we can derive the dependency between the QoS service exponent  $\theta$  and the constant arrival rate  $r$  following Equations (10) and (11). We obtain:

$$\theta = 2 \cdot \frac{m_s - r}{m_s \cdot (\lambda \cdot N - m_s)} = 2 \cdot \frac{\lambda \cdot N \cdot p_\lambda - r}{\lambda \cdot N \cdot p_\lambda \cdot (\lambda \cdot N - p_\lambda)}. \quad (15)$$

Combining Equations (15) and (11) yields finally:

$$\begin{aligned} r^* &= \frac{m_s}{2} \cdot \left( \sqrt{\frac{2 \cdot \ln(\mathcal{P}_d) \cdot (\lambda \cdot N - m_s)}{m_s \cdot d}} + 1 + 1 \right) \\ &= \frac{m_s}{2} \cdot \left( \sqrt{\frac{U}{p_\lambda} - U + 1 + 1} \right), \end{aligned} \quad (16)$$

where  $U = 2 \ln(\mathcal{P}_d) / d$ . Equation (16) gives us an upper bound on the arrival rate  $r$  per time frame. Recall the assumption

<sup>2</sup>The log-moment generating function of a Bernoulli process can also be determined directly. Numerically, this makes little difference.

that the transmitter always schedules a fixed data size of  $\lambda \cdot N$  bits to be transmitted. That raises the question how this choice impacts the maximum sustainable rate. Clearly, this scheduling decision should depend on the average channel state  $\bar{\gamma}$  as well as the QoS targets  $\mathcal{P}_d$  and  $d$ . We first establish a search range for the optimum. The upper bound is obtained from the argument of the square root in Equation (16) as:

$$\lambda^{\max} = \log_2 \left( 1 - \bar{\gamma} \cdot \beta \cdot \ln \left( \frac{\ln \mathcal{P}_d^2}{\ln \mathcal{P}_d^2 - d} \right) \right). \quad (17)$$

From numerical investigations, we conclude that within the range  $[0, \lambda^{\max}]$  there is an optimal setting for  $\lambda$ . However, the optimal setting for  $\lambda$  is hard to derive mathematically. Hence, we propose a simple search within the range  $[0, \lambda^{\max}]$  to determine a close-to-optimal setting for  $\lambda$ .

#### IV. EFFECTIVE SERVICE CAPACITY WITH PERFECT (INSTANTANEOUS) SNR

In this section we consider the performance when the transmitter has perfect, instantaneous channel state information. As a consequence, the transmitter knows exactly the transport capacity  $c_i$  of the upcoming transmission slot  $i$  and schedules exactly  $\sigma_i = c_i$  bits for transmission (if that amount of bits is available in the queue). Clearly, the service process  $s_i$  is no longer of Bernoulli type. Soret et al. have derived the corresponding expressions for the effective capacity in this case [7]. Adjusting to our notation, the mean and second moment of the service process have been reported to be:

$$\begin{aligned} \mathbb{E}[s_i] &= \frac{N}{\bar{\gamma}} \cdot \int_0^{\infty} \log_2(1 + \beta \cdot x) \cdot e^{-\frac{x}{\bar{\gamma}}} dx \\ &= \frac{N}{\ln(2)} \cdot e^{\frac{1}{\bar{\gamma}\beta}} \cdot E_1 \left( \frac{1}{\bar{\gamma}\beta} \right) \end{aligned} \quad (18)$$

$$\begin{aligned} \mathbb{E}[s_i^2] &= \frac{N^2}{\bar{\gamma}} \cdot \int_0^{\infty} (\log_2(1 + \beta \cdot x))^2 \cdot e^{-\frac{x}{\bar{\gamma}}} dx \\ &= \frac{N^2 e^{\frac{1}{\bar{\gamma}\beta}}}{\ln^2(2)} \left[ \left( C + \ln \left( \frac{1}{\bar{\gamma}\beta} \right) \right)^2 + \frac{\pi^2}{6} \right] \\ &\quad - \frac{N^2 e^{\frac{1}{\bar{\gamma}\beta}}}{\ln^2(2)} \frac{2}{\bar{\gamma}\beta} {}_3F_3 \left( 1, 1, 1; 2, 2, 2; -\frac{1}{\bar{\gamma}\beta} \right), \end{aligned} \quad (19)$$

where  $E_1(\cdot)$  is the exponential integral function,  $C$  is the Euler constant and  ${}_nF_m$  is the generalized hyper-geometric function. Given these expressions, we can obtain the variance of the process  $s_i$  and derive the maximum sustainable arrival rate of the source by Equation (11).

#### V. EFFECTIVE SERVICE CAPACITY WITH OUTDATED (INSTANTANEOUS) SNR AND SIMPLE SCHEDULING POLICIES

We now turn to the case that the transmitter has outdated instantaneous channel state information regarding the channel state of the upcoming transmission slot  $i$ . Denote this channel state information by  $\hat{\gamma}_i$ . In practice any channel state information is outdated and there are multiple alternatives

how the channel state information might be obtained by the transmitter (for example, obtaining  $\hat{\gamma}_i$  from an RTS/CTS handshake, from overhearing previous packet transmissions from the receiver, obtaining  $\hat{\gamma}_i$  from the acknowledgment frame, having an explicit feedback mechanism from the receiver to the transmitter etc.). We assume in the following that the channel state information  $\hat{\gamma}_i$  has been obtained by sampling the channel  $T_o$  time units prior to the upcoming transmission slot (where for the frame duration we have  $T_f \gg T_o$ ). Therefore,  $\hat{\gamma}_i$  and the real channel state  $\gamma_i$  are correlated. Let us denote the correlation coefficient of the two channel states by  $\rho^2$  where  $\rho$  is the correlation coefficient of either the inphase or quadrature Gaussian random process which both constitute the Rayleigh-distributed fading process with Jakes power spectrum. In general  $\rho$  depends on  $T_o$  as well as the maximum Doppler frequency in the propagation environment.

Given the channel estimate  $\hat{\gamma}_i$  the transmitter has to schedule now a packet size  $\sigma_i$  for the upcoming transmission slot. In the following we study simple scheduling policies where the scheduling decision is a function of the channel estimate, i.e.  $\sigma_i = N \cdot \log_2(1 + \beta \cdot \zeta^2 \cdot \hat{\gamma}_i)$ . Thus, the transmitter schedules data by including an SNR margin  $\zeta^2$  as the resulting service process yields  $s_i = N \cdot \log_2(1 + \beta \cdot \zeta^2 \cdot \hat{\gamma}_i)$  if the SNR  $\gamma_i$  during the upcoming transmission slot  $i$  is equal or above the threshold  $\hat{\gamma}_i \cdot \zeta^2$ . Therefore, if  $\zeta^2 \geq 1$  we call the scheduling scheme 'optimistic' whereas if  $\zeta^2 < 1$  we call the scheduling scheme 'pessimistic'. Note that we only consider scheduling policies for which the SNR margin is constant. More sophisticated scheduling schemes that adapt to the channel estimate or to the average channel state are left for future work.

In order to obtain the effective service capacity in this case, we initially consider the success probability for a scheduling decision  $\sigma_i = N \cdot \log_2(1 + \beta \cdot \zeta^2 \cdot \hat{\gamma}_i)$  under the condition that the correlation coefficient of the SNR estimate and the slot SNR is  $\rho^2$ . For a Rayleigh-fading process with Jakes power spectrum density the conditional probability density function  $p_{x|\hat{\gamma}}$  of the SNR of the upcoming transmission slot is given by [17]:

$$p_{x|\hat{\gamma}} = \frac{1}{\bar{\gamma} \cdot (1 - \rho^2)} \cdot I_0 \left( \frac{2 \cdot \rho \cdot \sqrt{x \cdot \hat{\gamma}}}{\bar{\gamma} \cdot (1 - \rho^2)} \right) \cdot e^{-\frac{x + \rho^2 \cdot \hat{\gamma}}{\bar{\gamma} \cdot (1 - \rho^2)}}, \quad (20)$$

where  $I_0$  is the modified Bessel function of the first kind. Hence, if the current channel estimate is  $\hat{\gamma}$  and the scheduled data amount equals  $N \cdot \log_2(1 + \beta \cdot \zeta^2 \cdot \hat{\gamma})$ , the success probability of the transmitted packet is given by:

$$\begin{aligned} \text{Pr.} \{s_i = N \cdot \log_2(1 + \beta \cdot \zeta^2 \cdot \hat{\gamma})\} &= \int_{\zeta^2 \cdot \hat{\gamma}}^{\infty} p_{x|\hat{\gamma}} \cdot dx \\ &= \int_{\zeta^2 \cdot \hat{\gamma}}^{\infty} \frac{1}{\bar{\gamma} \cdot (1 - \rho^2)} \cdot I_0 \left( \frac{2 \cdot \rho \cdot \sqrt{x \cdot \hat{\gamma}}}{\bar{\gamma} \cdot (1 - \rho^2)} \right) \cdot e^{-\frac{x + \rho^2 \cdot \hat{\gamma}}{\bar{\gamma} \cdot (1 - \rho^2)}} \cdot dx = \\ &= \int_{\frac{\sqrt{2 \cdot \hat{\gamma}}}{\sqrt{\bar{\gamma} \cdot (1 - \rho^2)}}}^{\infty} z \cdot I_0 \left( z \cdot \rho \cdot \frac{\sqrt{2 \cdot \hat{\gamma}}}{\sqrt{\bar{\gamma} \cdot (1 - \rho^2)}} \right) \cdot e^{-\frac{z^2}{2} - \frac{\rho^2 \cdot \hat{\gamma}}{\bar{\gamma} \cdot (1 - \rho^2)}} \cdot dz \end{aligned}$$

$$\begin{aligned}
&= \int_{\varsigma \cdot z_{\hat{\gamma}}}^{\infty} z \cdot I_0(z \cdot \rho \cdot z_{\hat{\gamma}}) \cdot e^{-\frac{z^2 + (\rho \cdot z_{\hat{\gamma}})^2}{2}} \cdot dz \\
&= Q(\rho \cdot z_{\hat{\gamma}}, \varsigma \cdot z_{\hat{\gamma}}), \quad (21)
\end{aligned}$$

where we substitute  $z = \sqrt{\frac{2 \cdot x}{\hat{\gamma} \cdot (1 - \rho^2)}}$  in the step from the second to the third line, and set  $\sqrt{\frac{2 \cdot \hat{\gamma}}{\hat{\gamma} \cdot (1 - \rho^2)}} = z_{\hat{\gamma}}$  in the fourth and fifth line. Finally,  $Q(a, b)$  is the Marcum Q-function.

We now turn to the problem of finding the mean and variance of the corresponding service process which allows us to derive the log-moment generating function according to Equation (8). We start with the derivation of the mean where we have:

$$E[s_i] = \frac{N}{\hat{\gamma}} \int_0^{\infty} \log_2(1 + \beta \varsigma^2 x) Q(\rho z_x, \varsigma z_x) e^{-\frac{x}{\hat{\gamma}}} dx \quad (22)$$

The derivation of this mean is quite involved due to the Marcum Q-function. We rely in the following on an upper bound for the Marcum Q-function  $Q(a, b)$  which is based on a geometric approach [18]. For the case that  $a < b$  the bound is as follows:

$$Q(a, b) \leq \frac{1}{\pi} \sum_{i=0}^{\Theta-1} e^{-\frac{(\sqrt{b^2 - a^2 \sin^2 \theta_i} - a \cos \theta_i)^2}{2}} \cdot (\theta_{i+1} - \theta_i) \quad (23)$$

in which the set of  $\theta_i \in [0; \pi]$  can be chosen arbitrarily and  $\Theta \in \mathbb{N}$  determines the accuracy of the bound. In the following we set  $\theta_i = \frac{\pi}{\Theta} \cdot i$ . The condition  $a < b$ , which is fulfilled as long as the transmitter chooses  $\varsigma \geq \rho$ , is a significant restriction for the scheduling policy especially if the correlation  $\rho$  is high. Hence, for the case that  $a \geq b$  we use as upper bound [18]:

$$\begin{aligned}
Q(a, b) &\leq 1 - \\
&\frac{1}{\pi} \sum_{i=0}^{\Theta-1} e^{-\frac{(-\sqrt{b^2 - a^2 \sin^2 \theta_{i+1}} + a \cos \theta_{i+1})^2}{2}} \cdot (\theta_{i+1} - \theta_i) \\
&+ \frac{1}{\pi} \sum_{i=0}^{\Theta-1} e^{-\frac{(\sqrt{b^2 - a^2 \sin^2 \theta_{i+1}} + a \cos \theta_{i+1})^2}{2}} \cdot (\theta_{i+1} - \theta_i) \quad (24)
\end{aligned}$$

where in this case we have  $\theta_i \in [0; \theta_{\max}]$  with  $\theta_{\max} = \arcsin \frac{b}{a}$ . Again we choose in the following a regular sampling of the angle with  $\theta_i = \frac{\theta_{\max}}{\Theta} \cdot i$ .

Considering the arguments of the Marcum Q-function in Equation (22), we obtain for the upper bound in the case that  $\rho < \varsigma$  from Equation (23):

$$Q(\rho z_x, \varsigma z_x) \leq \sum_{i=0}^{\Theta-1} K_i \cdot e^{-\frac{x}{\hat{\gamma}} \cdot a_i^{(1)}}. \quad (25)$$

Correspondingly, for the case that  $\rho \geq \varsigma$  we obtain from Equation (24):

$$Q(\rho z_x, \varsigma z_x) \leq 1 - \sum_{i=0}^{\Theta-1} K_i e^{-\frac{x}{\hat{\gamma}} \cdot a_i^{(2)}} + \sum_{i=0}^{\Theta-1} K_i e^{-\frac{x}{\hat{\gamma}} \cdot a_i^{(3)}}. \quad (26)$$

In the two equations above we set:

$$\begin{aligned}
K_i &= \frac{\theta_{i+1} - \theta_i}{\pi} \\
a_i^{(1)} &= \frac{(\sqrt{\varsigma^2 - \rho^2} \sin^2 \theta_i - \rho \cos \theta_i)^2}{(1 - \rho^2)} \\
a_i^{(2)} &= \frac{(-\sqrt{\varsigma^2 - \rho^2} \sin^2 \theta_{i+1} + \rho \cos \theta_{i+1})^2}{(1 - \rho^2)} \\
a_i^{(3)} &= \frac{(\sqrt{\varsigma^2 - \rho^2} \sin^2 \theta_{i+1} + \rho \cos \theta_{i+1})^2}{(1 - \rho^2)} \quad (27)
\end{aligned}$$

Based on these expressions for the Marcum Q-function, we can now proceed to approximate the mean of the service process obtained from scheduling based on outdated channel state information by substituting Equations (25) and (26) for the Marcum Q-function in Equation (22). Based on Equation (31) from the Appendix, we obtain therefore for the mean of Equation (22) if  $\rho < \varsigma$ :

$$\begin{aligned}
E[s_i] &= \frac{N}{\hat{\gamma}} \cdot \int_0^{\infty} \log_2(1 + \beta \varsigma^2 x) Q_1(\rho z_x, \varsigma z_x) e^{-\frac{x}{\hat{\gamma}}} dx \\
&\leq N \cdot \sum_{i=0}^{\Theta-1} \int_0^{\infty} \frac{K_i}{\hat{\gamma}} \log_2(1 + \beta \varsigma^2 x) \cdot e^{-\frac{x}{\hat{\gamma}} (1 + a_i^{(1)})} dx \\
&= \sum_{i=0}^{\Theta-1} \frac{N \cdot K_i}{(1 + a_i^{(1)}) \cdot \ln 2} \cdot e^{\frac{1 + a_i^{(1)}}{\beta \cdot \varsigma^2 \cdot \hat{\gamma}}} \cdot E_1 \left( \frac{1 + a_i^{(1)}}{\beta \cdot \varsigma^2 \cdot \hat{\gamma}} \right), \quad (28)
\end{aligned}$$

In a similar manner we obtain the mean of the service process in case that  $\rho \geq \varsigma$ .

Finally, we turn to the derivation of the second moment, for which we obtain for the case of  $\rho < \varsigma$  based on Equation (25) and on Equation (41) from the Appendix:

$$\begin{aligned}
E[s_i^2] &= \frac{N^2}{\hat{\gamma}} \cdot \int_0^{\infty} (\log_2(1 + \beta \varsigma^2 x))^2 Q_1(\rho z_x, \varsigma z_x) e^{-\frac{x}{\hat{\gamma}}} dx \\
&\leq \sum_{i=0}^{\Theta-1} \int_0^{\infty} \frac{K_i N^2}{\hat{\gamma}} (\log_2(1 + \beta \varsigma^2 x))^2 \cdot e^{-\frac{x}{\hat{\gamma}} (1 + a_i^{(1)})} dx = \\
&\sum_{i=0}^{\Theta-1} \frac{K_i N^2}{(1 + a_i^{(1)}) \ln^2(2)} e^{\frac{1 + a_i^{(1)}}{\beta \varsigma^2 \hat{\gamma}}} \left[ \left( C + \ln \left( \frac{1 + a_i^{(1)}}{\beta \varsigma^2 \hat{\gamma}} \right) \right)^2 + \frac{\pi^2}{6} \right] - \\
&\sum_{i=0}^{\Theta-1} \frac{2 K_i N^2}{\beta \varsigma^2 \hat{\gamma} \ln^2(2)} e^{\frac{1 + a_i^{(1)}}{\beta \varsigma^2 \hat{\gamma}}} {}_3F_3 \left( 1, 1, 1; 2, 2, 2; -\frac{1 + a_i^{(1)}}{\beta \varsigma^2 \hat{\gamma}} \right). \quad (29)
\end{aligned}$$

The case  $\rho \geq \varsigma$  is obtained in a similar manner based on Equation (26) and Equation (41) from the Appendix. Finally, to obtain the effective service capacity, we derive from Equations (28) and (29) the variance  $\text{Var}[s_i]$  of the service process. Then, based on Equation (11), we can obtain the maximum sustainable arrival rate for the specified QoS constraints. Note that this rate is a function of  $\varsigma$  and, hence, of the degree of 'optimism' in the scheduling at the transmitter. Furthermore, depending on the relationship between  $\rho$  and  $\varsigma$  the correct formulas need to be chosen.

## VI. NUMERICAL EVALUATION

In this section we evaluate our analytical expressions numerically. We pursue two different goals. On the one hand, we have used several approximations and bounds for deriving

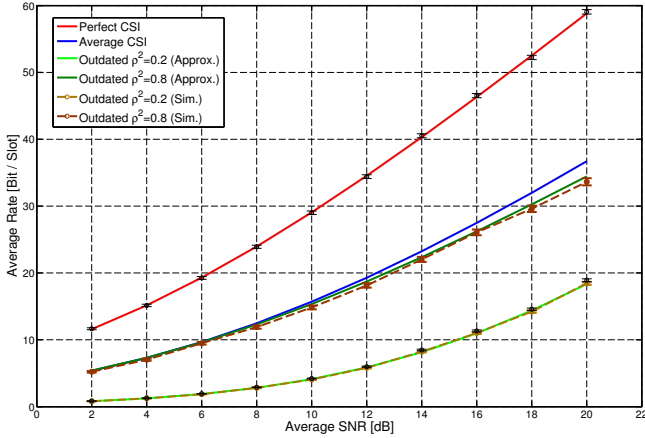


Figure 1. Comparison of the analytically derived average rate of various schemes versus simulation results for various average SNR settings (10 symbols slot length, SNR margin for outdated CSI cases set to  $\zeta^2 = 1$ ).

the effective service capacity for the outdated CSI case. Hence, we are interested in validating the analysis by simulations. On the other hand, our results allow us to investigate the optimal SNR margin  $\zeta$  if data scheduling is based on outdated CSI. This is our second goal.

We start with the validation which is done by means of simulations. The simulation is based on generating sequences of exponentially distributed SNR values, converting these values into service units and then simulating (and observing) the resulting behavior a queue for the various different system schemes under study. In Figure 1 we initially consider the average rate of the different system designs over an increasing SNR. The plot contains the results for the perfect CSI case (including simulation results indicated by the confidence intervals), the average CSI case (not containing any simulation results) as well as the results for scheduling with outdated CSI (for  $\rho = 0.2$  and  $\rho = 0.8$  and setting  $\zeta^2 = 1$ ). In the case of using outdated CSI we also present curves for the corresponding simulations results. Regarding the validation, the results on the average rate in case of outdated CSI indicate that the used bound from Equation 23 indeed provides analytical results which are very close to the real system behavior as obtained from simulations. Apart from this validation result, the figure also reveals already some interesting system behavior. First notice that the average rate for scheduling with outdated CSI is in both cases worse than the corresponding result from scheduling with average CSI only. Furthermore, the difference of these two schemes to the case of scheduling with perfect CSI is quite large (usually about 50%).

Next, we present in Figure 2 and 3 the maximum sustainable rate resulting from analysis and simulations for the different considered system designs. In both cases we consider the following parameters: 10 dB average SNR, delay target of 10 slots and outage probability of 0.1. In addition, we vary the SNR margin  $\zeta^2$  between 0.01 and 1. First of all, the two figures demonstrate nicely the correspondence between the analytical results and simulations for the maximum sustainable rate. Notice that this validation includes now not only the

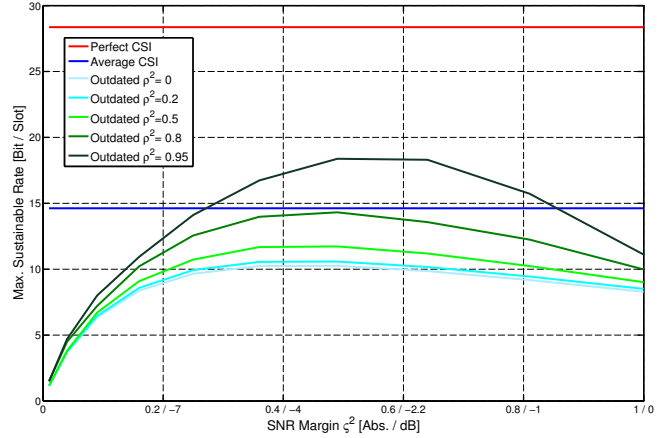


Figure 2. Analytical results of the maximum sustainable rate of various scheduling schemes for the outdated CSI case (10 symbols slot length, 10 dB average SNR, target delay of 10 slots, outage probability of 0.1). Results on scheduling with outdated CSI are compared to the ones of scheduling with perfect CSI and (fixed) scheduling with average CSI only.

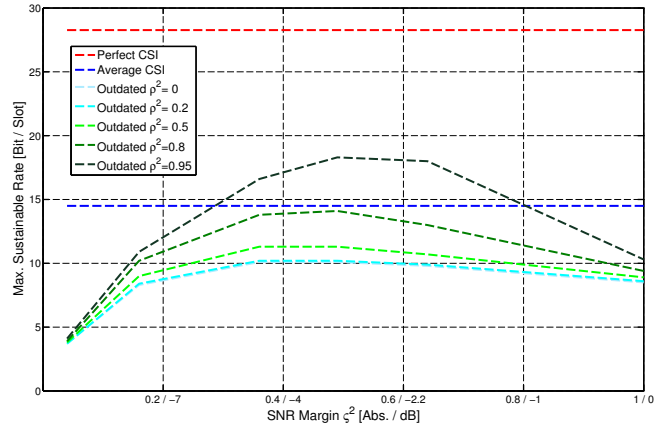


Figure 3. Simulation results of the maximum sustainable rate from various scheduling schemes for the outdated CSI case (10 symbols slot length, 10 dB average SNR, target delay of 10 slots, outage probability of 0.1). Results on scheduling with outdated CSI are compared to the ones of scheduling with perfect CSI and (fixed) scheduling with average CSI only.

upper bound for the Marcum Q-function but also the Gaussian approximation for the derivation of the log-moment generating function from Soret et al. in Equation 11. In addition to this important validation aspect, the figures also reveal a significant performance behavior for the case of scheduling with outdated CSI. As the SNR margin increases from 0.01 to 1, the maximum rate first increases, reaches an optimum and decreases afterwards. For all considered cases of the correlation strength  $\rho$  the optimum SNR margin is close to 0.5 equaling a 3 dB margin (the lower the correlation, the lower the optimal SNR margin is in tendency). Notice in particular that even for a strong correlation between the channel estimate and the slot channel state of 0.95, the maximum sustainable rate is only slightly bigger than the one obtained by scheduling based on average CSI only. Furthermore, optimistic scheduling (with an SNR margin of 1) achieves a significantly lower maximum sustainable rate than using average CSI only.

Finally, in Figure 4 and 5 we study the behavior of the SNR

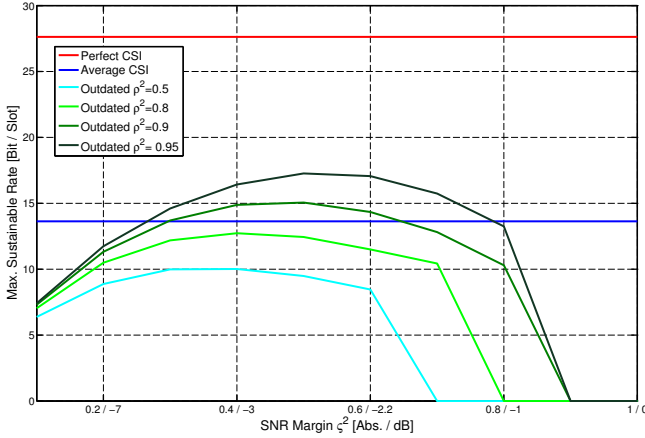


Figure 4. Analytical results of the maximum sustainable rate of various scheduling schemes for the outdated CSI case (10 symbols slot length, 10 dB average SNR, target delay of 10 slots, outage probability of 0.01). Results on scheduling with outdated CSI are compared to the ones of scheduling with perfect CSI and (fixed) scheduling with average CSI only.

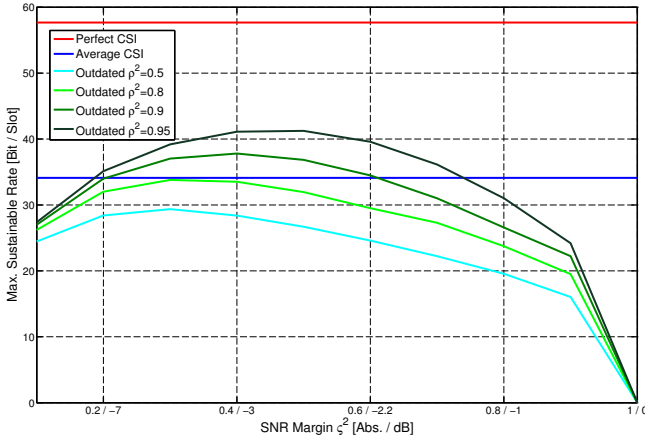


Figure 5. Analytical results of the maximum sustainable rate of various scheduling schemes for the outdated CSI case (10 symbols slot length, 20 dB average SNR, target delay of 10 slots, outage probability of 0.01). Results on scheduling with outdated CSI are compared to the ones of scheduling with perfect CSI and (fixed) scheduling with average CSI only.

margin as the QoS requirements become more restrictive as well as the average SNR increases. In Figure 4 we consider a target outage probability of 0.01 (instead of 0.1 as in the previous case). This leads to a lower maximum sustainable rate in general. For the case of scheduling with outdated CSI the optimal SNR margins become smaller. Hence, a more pessimistic data scheduling is beneficial. This tendency is also present if we increase the average SNR of the channel to 20 dB, as studied in Figure 5. In general, this increases the maximum sustainable rate for all studied schemes. However, the optimal SNR margins for scheduling with outdated CSI are furthermore decreased. Notice in both Figures 4 and 5 that optimistic scheduling can not support any arrival rate with the required QoS targets.

## VII. CONCLUSIONS

In this paper we have considered the effective service capacity of scheduling data frames based on outdated CSI

information. In contrast to previous work, we focus on the dependency between the scheduling approach at the transmitter and resulting loss of the packet transmission due to overestimating the upcoming channel state. Based on the mathematical expressions for effective service capacity when scheduling with outdated CSI, we can show that there exists an optimal SNR margin that increases the maximum sustainable rate of a given scenario. This margin is typically around 3 dB. Furthermore, we find that adapting to the channel state only pays off if the channel estimate is highly correlated to the channel state (at least a coefficient of 0.9). Otherwise, a fixed frame size strategy without adapting to the instantaneous channel state is more advantageous. Finally, we show that the optimal SNR margin is sensitive to the QoS target as well as to the average channel state. As the QoS targets become tougher and/or the average channel state increases the optimal SNR margin increases, i.e. more pessimistic scheduling is beneficial.

There are plenty of open issues for further study. Among them are the consideration of interference-limited channels, more sophisticated scheduling schemes for the outdated CSI case, including the impact of hybrid ARQ on the here presented model and the consideration of multi-user scheduling.

## APPENDIX

### A. First Moment of the Capacity of a Scaled Exponential Random Variable

In this section we deal with the derivation of the following integral:

$$\int_0^{\infty} \frac{K}{c} \cdot \log_2(1 + b \cdot x) \cdot e^{-\frac{a \cdot x}{c}} dx. \quad (30)$$

We obtain based on Table of Integrals [19]:

$$\begin{aligned} & \int_0^{\infty} \frac{K}{c} \cdot \log_2(1 + b \cdot x) \cdot e^{-\frac{a \cdot x}{c}} dx = \\ & -\frac{K}{a \cdot \ln 2} \cdot \left[ e^{\frac{a}{bc}} \cdot E_1\left(\frac{bxa+a}{bc}\right) + e^{-\frac{ax}{c}} \cdot \ln(bx + 1) \right]_0^{\infty} \\ & = \frac{K}{a \cdot \ln 2} \cdot e^{\frac{a}{bc}} \cdot E_1\left(\frac{a}{bc}\right), \end{aligned} \quad (31)$$

which is a straight-forward extension of the well known result for  $a = 1$ . Above,  $E_1(x) = \int_1^{\infty} \frac{e^{-x \cdot t}}{t^n} dt$  is the exponential integral function.

### B. Second Moment of the Capacity of a Scaled Exponential Random Variable

Next, we consider the computation of the second moment of the scaled exponential random variable. Formally, we are interested in:

$$\int_0^{\infty} \frac{K}{c} \cdot (\log_2(1 + b \cdot x))^2 \cdot e^{-\frac{a \cdot x}{c}} dx. \quad (32)$$

By simple substitution and manipulation (we change Equation (32) to:

$$\frac{K}{bc \ln^2(2)} \int_0^{\infty} \ln^2(1 + z) e^{-sz} dz = \frac{K}{bc \ln(2)} \cdot \mathcal{L}\{\ln^2(1 + z)\}, \quad (33)$$



where  $s = \frac{a}{b \cdot c}$  and  $\mathcal{L}\{f\}$  denotes the Laplace transform of  $f$ . Initially, based on this interpretation, there is no straightforward solution for this transform. However, notice the following identity for Laplace transforms:  $\mathcal{L}\{f(x+t)\} = e^{s \cdot t} \cdot \left[ \mathcal{L}\{f(x)\} - \int_0^t f(x) \cdot e^{-sx} dx \right]$ . Furthermore, we have  $\mathcal{L}\left\{\ln^2(x) - \frac{\pi^2}{6}\right\} = \frac{(C+\ln(s))^2}{s}$ , where  $C$  is the Euler constant. Applying both results to Equation (33) yields:

$$\frac{K e^s}{bc \ln^2(2)} \left[ \frac{(C + \ln(s))^2}{s} + \frac{\pi^2}{6 \cdot s} - \int_0^1 \ln^2(z) e^{-sz} dz \right]. \quad (34)$$

Let us focus on the integrand in Equation (34). We obtain:

$$\begin{aligned} \int_0^1 \ln^2(z) e^{-sz} dz &= 2z {}_3F_3(1, 1, 1; 2, 2, 2; -sz) - \frac{\ln(z)}{s} \cdot \\ &\left( (e^{-sz} - 1) \ln(z) + 2(\ln(sz) + C) + 2\Gamma(0, sz) \right) \Big|_0^1 \\ &= 2 {}_3F_3(1, 1, 1; 2, 2, 2; -s) + \lim_{z \rightarrow 0} \frac{\ln(z)}{s} \cdot \\ &\left( (e^{-sz} - 1) \ln(z) + 2(\ln(sz) + C) + 2\Gamma(0, sz) \right). \quad (35) \end{aligned}$$

Above,  $\Gamma(\cdot)$  is the incomplete gamma function and  ${}_3F_3(\cdot)$  is the generalized hypergeometric function. For the first product of the limit we obtain by application of l'Hospital's rule:

$$\begin{aligned} \lim_{z \rightarrow 0} \frac{e^{-sz} - 1}{\ln^{-2}(z)} &= \lim_{z \rightarrow 0} \frac{se^{-sz}}{2z^{-1} \ln^{-3}(z)} = \frac{s}{2} \lim_{z \rightarrow 0} \frac{\ln^3(z)}{z^{-1}} \\ \frac{s}{2} \lim_{z \rightarrow 0} \frac{3 \ln^2(z)}{-z^{-1}} &= \frac{s}{2} \lim_{z \rightarrow 0} \frac{6 \ln(z)}{z^{-1}} = \frac{s}{2} \lim_{z \rightarrow 0} 6z = 0. \quad (36) \end{aligned}$$

Now we consider the limit of the second and third product in Equation (35). We initially obtain:

$$\frac{2}{s} \lim_{z \rightarrow 0} \frac{\ln(sz) + C + \Gamma(0, sz)}{\ln^{-1}(z)} = \frac{2}{s} \lim_{z \rightarrow 0} \frac{1 + \frac{C + \Gamma(0, sz)}{\ln(sz)}}{\ln^{-1}(sz) \cdot \ln^{-1}(z)}. \quad (37)$$

We first focus on the numerator and obtain:

$$\lim_{z \rightarrow 0} 1 + \frac{C + \Gamma(0, sz)}{\ln(sz)} = 1 + \lim_{z \rightarrow 0} \frac{(-s)e^{-sz}(sz)^{-1}}{s(sz)^{-1}} = \lim_{z \rightarrow 0} 1 - e^{-sz}. \quad (38)$$

Hence, the limit of Equation (37) equals:

$$\lim_{z \rightarrow 0} \frac{1 - e^{-sz}}{\ln^{-1}(sz) \cdot \ln^{-1}(z)} = 0, \quad (39)$$

according to the result of Equation (36).

Therefore, we have:

$$\int_0^1 \ln^2(z) e^{-sz} dz = 2 {}_3F_3(1, 1, 1; 2, 2, 2; -s), \quad (40)$$

and this yields finally:

$$\begin{aligned} &\int_0^\infty \frac{K}{c} \cdot (\log_2(1 + b \cdot x))^2 \cdot e^{-\frac{a \cdot x}{c}} dx = \\ &\frac{K e^s}{bc \ln^2(2)} \left[ \frac{(C + \ln(s))^2}{s} + \frac{\pi^2}{6 \cdot s} - 2 {}_3F_3(1, 1, 1; 2, 2, 2; -s) \right] \\ &= \frac{K e^{\frac{a}{bc}}}{a \ln^2(2)} \left[ \left( C + \ln\left(\frac{a}{bc}\right) \right)^2 + \frac{\pi^2}{6} \right] - \\ &\frac{2K e^{\frac{a}{bc}}}{bc \ln^2(2)} {}_3F_3\left(1, 1, 1; 2, 2, 2; -\frac{a}{bc}\right). \quad (41) \end{aligned}$$

## REFERENCES

- [1] I. Hou, V. Borkar, and P. Kumar, "A theory of QoS for wireless," in *Proc. of the IEEE International Conference on Computer Communications (Infocom)*, Rio de Janeiro, Brazil, Apr. 2009.
- [2] M. Fidler, "A network calculus approach to probabilistic quality of service analysis of fading channels," in *Proc. of the IEEE Global Telecommunications Conference (Globecom)*, San Francisco, USA, Dec. 2006.
- [3] —, "Survey of deterministic and stochastic service curve models in the network calculus," *IEEE Communications Surveys & Tutorials*, vol. 12, no. 1, pp. 59–86, 2010.
- [4] F. Ciucu, J. Schmitt, and W. Hao, "On expressing networks with flow transformations in convolution-form," in *Proc. of the IEEE International Conference on Computer Communications (Infocom)*, Shanghai, China, Apr. 2011.
- [5] D. Wu and R. Negi, "Effective capacity: a wireless link model for support of quality of service," *IEEE Trans. Wireless Commun.*, vol. 2, no. 4, pp. 630–643, 2003.
- [6] L. Lingjia and J. Chamberland, "On the effective capacities of multiple-antenna gaussian channels," in *Proc. of the IEEE International Symposium on Information Theory (ISIT)*, Toronto, Canada, Jul. 2008.
- [7] B. Soret, M. C. Aguayo-Torres, and J. T. Entrambasaguas, "Capacity with Explicit Delay Guarantees for Generic Sources over Correlated Rayleigh Channel," *IEEE Trans. Wireless Commun.*, vol. 9, no. 6, pp. 1901–1911, 2010.
- [8] Y. Xi, A. Burr, J. Bo, and D. Grace, "A General Upper Bound to Evaluate Packet Error Rate over Quasi-Static Fading Channels," *IEEE Trans. Wireless Commun.*, vol. 10, no. 5, pp. 1373–1377, 2011.
- [9] C. Chang, "Stability, queue length and delay of deterministic and stochastic queueing networks," *IEEE Trans. Automat. Contr.*, vol. 39, no. 5, pp. 913–931, 1994.
- [10] A. Kumar, D. Manjunath, and J. Kuri, *Communication Networking: An Analytical Approach*. Morgan Kaufmann, 2004.
- [11] D. Wu and R. Negi, "Effective capacity channel model for frequency-selective fading channels," in *Proc. of the ICST Second International Conference on Quality of Service in Heterogeneous Wired/Wireless Networks*, Orlando, USA, Aug. 2005.
- [12] —, "Utilizing multiuser diversity for efficient support of quality of service over a fading channel," *IEEE Trans. Veh. Technol.*, vol. 54, no. 3, pp. 1198–1206, 2005.
- [13] —, "Downlink scheduling in a cellular network for quality-of-service assurance," *IEEE Trans. Veh. Technol.*, vol. 53, no. 5, pp. 1547–1557, 2004.
- [14] T. Jia and Z. Xi, "QoS-Driven Adaptive Power and Rate Allocation for Multichannel Communications in Mobile Wireless Networks," in *Proc. of the IEEE International Symposium on Information Theory (ISIT)*, Seattle, USA, Jul. 2006.
- [15] Q. Deli, M. GURSOY, and S. Velipasalar, "The impact of qos constraints on the energy efficiency of fixed-rate wireless transmissions," *IEEE Trans. Wireless Commun.*, vol. 8, no. 12, pp. 5957–5969, 2009.
- [16] G. Femenias, J. Ramis, and L. Carrasco, "Using two-dimensional markov models and the effective-capacity approach for cross-layer design in amc/arq-based wireless networks," *IEEE Trans. Veh. Technol.*, vol. 58, no. 8, pp. 4193–4203, 2009.
- [17] R. Mallik, "On multivariate Rayleigh and exponential distributions," *IEEE Trans. Inform. Theory*, vol. 49, no. 6, pp. 1499–1515, 2003.
- [18] N. Ding and H. Zhang, "A Flexible Method to Approximate Marcum Q-Function Based on Geometric Way of Thinking," in *Proc. of the International Symposium on Communications, Control and Signal Processing (ISCCSP)*, Malta, Mar. 2008.
- [19] A. Prudnikov, Y. Brychkov, and O. Marichev, *Integrals and Series: Volume 1*. Gordon and Breach Science Publishers, 1986.