

Relaying with Finite Blocklength: Challenge vs. Opportunity

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Abstract—In this paper, we study the performance of a system with multiple decode-and-forward (DF) relays under the finite blocklength (FBL) regime. We derive the FBL-Throughput under both perfect CSI and average CSI scenarios while the corresponding throughputs under an infinite blocklength assumption (IBL-throughput) are discussed as performance references. Through numerical analysis, we evaluate the system performance. We show a higher throughput under the FBL assumption than under the IBL assumption under the perfect CSI scenario.

Index Terms—Finite blocklength regime, decode-and-forward, relaying, throughput, perfect CSI, average CSI

I. INTRODUCTION

Relaying [1], [2] is well known as an efficient way to greatly enhance the performance of wireless transmission by exploiting spatial diversity. Specifically, two-hop decode-and-forward (DF) relaying protocols significantly improve the throughput/capacity [3]–[6]. However, all the above studies of the advantages of relaying are based on an ideal assumption that the transmission is error-free or arbitrarily reliably at Shannon’s channel capacity where coding is assumed to be performed in the infinite blocklength (IBL) regime (using a code block of infinite length). Unfortunately, in practice it is impossible for systems to have infinite blocklengths.

If the codeword is restricted to a reasonable size, i.e., to a finite blocklength, the error probability of the communication becomes no longer arbitrarily small. Hence, in the finite blocklength (FBL) regime, it is essential to consider the error probability while investigating the communication performance. Recently, an accurate approximation of achievable coding rate was presented in [7] for a single-hop transmission system which also takes a block error probability (due to noise) into account. In [7] the authors show that the performance loss due to a finite blocklength is considerable and increases for a decreasing blocklengths. Subsequently, this fundamental study regarding additive white Gaussian noise (AWGN) channels was extended to Gilbert-Elliott channels [8], quasi-static fading channels [9], [10], quasi-static fading channels with retransmissions [11] as well as spectrum sharing networks [12]. However, all these works focus on single-hop non-relaying systems.

In a two-hop DF relaying network, relaying exploits spatial diversity but at the same time halves the blocklength of the transmission (if equal time division is considered). As the performance loss increases under the

FBL model as the blocklength decreases [7], an interesting question arises: Does relaying pay off less in the FBL regime? In our previous work [13]–[15], we address these questions under a single relay scenario where we find that the performance loss (due to FBL) in the case of relaying is much smaller than expected, while the performance loss of direct transmission is larger. In other words, under the FBL model relaying is in fact more beneficial as the FBL effect due to the shortening of the time frame (in case of relaying) is more or less compensated by higher channel quality at each hop. In this paper, we extend the study to a multi-DF-relay scenario where the information-theoretic performance limit is not known. Both perfect CSI and average CSI (at the transmitter) assumptions are considered in this work.

The rest of the paper is organized as follows. Section II describes the system model and briefly introduces the background theory regarding the FBL regime. In Section III and IV, we derive the blocklength-limited performance of the multi-relay system under perfect CSI and average CSI scenarios while the corresponding throughputs under the IBL regime are also discussed as comparison schemes. Section V presents our numerical results. Finally, we conclude our work in Section VI.

II. PRELIMINARIES

A. System Model

We consider a simple scenario with a source S, a destination D and J DF relays $R_j, j = 1, 2, \dots, J$ as schematically shown in Fig. 1. In general, the distances between relays are assumed to be significantly shorter than the distances either from the source to relays or from the relays to the destination. The entire system operates in

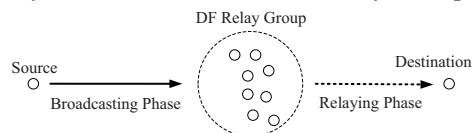


Fig. 1. Example of the considered multi-relay system.

a slotted fashion where time is divided into transmission periods of length $2m$ (symbols). Each transmission period contains two phases (each phase with length m), which are referred to as broadcasting phase and relaying phase. In the broadcasting phase, the source sends a data block to relays. Afterwards, if at least one relay decodes the block successfully, all these relays forward the block

together to the destination in the subsequent relaying phase. As a result, the destination likely receives multiple coherent signal copies.

Channels are assumed to experience Rayleigh block-fading, i.e., channels are constant during the duration of each transmission period but vary from one period to the next. The CSI of a link is assumed to be perfectly estimated at the receiver. We denote the channel gains from the source to the destination, from the source to relay j and from relay j to the destination during period i by $|h_{S,D,i}|^2$, $|h_{S,j,i}|^2$ and $|h_{j,D,i}|^2$ ($j=1, 2, \dots, J$). The corresponding average channel gains are $|\bar{h}_{S,D}|^2$, $|\bar{h}_{S,j}|^2$ and $|\bar{h}_{j,D}|^2$. In addition, we denote P_{tx} as the transmit power at the source and each relay. The noise power is denoted by σ^2 . Also, we assume no interference to be present. Hence, the instantaneous signal-to-noise ratio (SNR) in transmission period i from the source to relay j is $\gamma_{S,j,i} = P_{\text{tx}}|h_{S,j,i}|^2/\sigma^2$. Similarly, the SNR of the link from relay j to the destination is $\gamma_{j,D,i} = P_{\text{tx}}|h_{j,D,i}|^2/\sigma^2$. As multiple relays forward the same data block during a relaying phase, the destination receives multiple signals of the block. By applying maximum ratio combining (MRC) during the reception of these signals, the destination obtains a joint instantaneous SNR as the sum of the instantaneous SNRs of links from these relays as $\gamma_{\xi_i,D,i} = \sum_{R_j \in \xi_i} \gamma_{j,D,i}$, where ξ_i is the set of relays which forward the packet during transmission period i .

B. FBL Performance of a Single-Hop Transmission

For AWGN channels, [7] derives a tight bound for the coding rate of a single-hop transmission system. With blocklength m , block error probability ε and SNR γ , the coding rate (in bits per channel use) is given by: $r = \frac{1}{2} \log_2(1 + \gamma) - \sqrt{(1 - \frac{1}{(1+\gamma)/2m^2})} Q^{-1}(\varepsilon) \log_2 e + \frac{O(\log_2 m)}{m}$, where $Q^{-1}(\cdot)$ is the inverse of the Q-function given by $Q(w) = \int_w^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$. In [10], the above result has been extended to a complex channel model with received SNR γ , where the coding rate (in bits per channel use) is:

$$r = \mathcal{R}(\gamma, \varepsilon, m) \approx \mathcal{C}(\gamma) - \sqrt{\frac{V}{m}} Q^{-1}(\varepsilon), \quad (1)$$

where $\mathcal{C}(\gamma)$ is the Shannon capacity. For a known SNR of the channel, it is given by $\mathcal{C}(\gamma) = \log_2(1 + \gamma)$. Moreover, V is the channel dispersion [7, Def.1]. Hence, for a single hop transmission with blocklength m and coding rate r , the decoding (block) error probability at the receiver is given by:

$$\varepsilon = \mathcal{P}(\gamma, r, m) = Q\left(\sqrt{\frac{m}{V}} (\mathcal{C}(\gamma) - r)\right). \quad (2)$$

Finally, the blocklength-limited throughput (FBL-throughput) C_{FBL} of the transmission is defined as the average/expected effectively transmitted information (the

number of correctly received bits at the destination) per channel use, given by: $C_{\text{FBL}} = (1 - \varepsilon)r$.

So far, we have introduced the system model and the performance model of a single-hop transmission with FBL. In the following, we will study the performance of the considered multi-relay system under two different assumptions: Having perfect CSI of all links at the source as well as having only average CSI of all links at the source. Both the infinite blocklength (IBL) regime and the FBL regime will be considered.

III. PERFECT CSI SCENARIO: IBL-THROUGHPUT VS. FBL-THROUGHPUT

A. The IBL-Throughput of Multi-Relay with Perfect CSI

In the IBL regime, with perfect CSI the source knows which relays will decode the data packet successfully for a given coding rate, i.e., by comparing the coding rate with the Shannon capacity of the link to the relay. Therefore, the source is able to determine an optimal coding rate to have an appropriate set of forwarding relays to maximize the throughput under the IBL regime. Denote the throughput at transmission period i of the considered two-hop multi-relay network by $C_{\text{IBL},i}$. $C_{\text{IBL},i}$ is actually the achievable/maximal throughput of the considered two-hop multi-relay system, and it is given by: $C_{\text{IBL},i} = \max_{\xi_i \in \mathbb{P}(S)} \{C_{\xi_i}\}$, where $\mathbb{P}(S)$ is the powerset of relay set $S = \{1, 2, \dots, J\}$ and C_{ξ_i} is the throughput of a multi-relay transmission with the forwarding relay set ξ_i . By combining these signals from all relays in ξ_i to the destination, the combined SNR at the destination is given by $\sum_{j \in \xi_i} \gamma_{j,D,i}$. Therefore, we have:

$$\begin{aligned} C_{\xi_i} &= \frac{1}{2} \min \left\{ \min_{j \in \xi_i} \{\mathcal{C}(\gamma_{S,j,i})\}, \mathcal{C} \left(\sum_{j \in \xi_i} \gamma_{j,D,i} \right) \right\} \\ &= \frac{1}{2} \mathcal{C} \left(\min_{j \in \xi_i} \left\{ \gamma_{S,j,i}, \sum_{k \in \xi_i} \gamma_{k,D,i} \right\} \right). \end{aligned} \quad (3)$$

Thus, the throughput of transmission period i is given by:

$$C_{\text{IBL},i} = \frac{1}{2} \mathcal{C} \left(\max_{\xi_i \in \mathbb{P}(S)} \left\{ \min_{j \in \xi_i} \left\{ \gamma_{S,j,i}, \sum_{k \in \xi_i} \gamma_{k,D,i} \right\} \right\} \right). \quad (4)$$

Finally, the IBL-Throughput of multi-relay with perfect CSI is given by the expectation of $C_{\text{IBL},i}$ over all possible realizations of the channel SNR: $C_{\text{IBL}}^{\text{perf}} = \mathbb{E}_{\gamma} [C_{\text{IBL},i}]$.

B. The FBL-Throughput of Multi-Relay with Perfect CSI

In the FBL regime, due to perfect instantaneous CSI the source is able to determine an appropriate coding rate for each transmission period. Denote the determined coding rate for transmission period i by r_i . According to (2) the error probability of the link from the source to relay j is given by: $\varepsilon_{S,j,i} = \mathcal{P}(\gamma_{S,j,i}, r_i, m)$. In other words, with

probability $1 - \varepsilon_{S,j,i}$ the relay j will decode the packet correctly and join the relaying phase. Recall that for a forwarding relay set ξ_i the combined SNR at the destination is given by $\sum_{j \in \xi_i} \gamma_{j,D,i}$. Then the error probability of the second hop is given by $\mathcal{P}\left(\sum_{j \in \xi_i} \gamma_{j,D,i}, r_i, m\right)$. Hence, the expected overall error probability of two-hop relaying at period i is:

$$\varepsilon_{MR,i} = \sum_{\xi_i \in \mathbb{P}(S)} \left\{ \prod_{j \notin \xi_i} \varepsilon_{S,j,i} \prod_{n \in \xi_i} (1 - \varepsilon_{S,n,i}) \mathcal{P}\left(\sum_{n \in \xi_i} \gamma_{n,D,i}, r_i, m\right) \right\}. \quad (5)$$

Therefore, with perfect CSI the (expected) FBL-throughput of transmission period i is given by:

$$C_{FBL,i} = (1 - \varepsilon_{MR,i})r_i/2. \quad (6)$$

Finally, the average FBL-throughput (over all possible realizations of the channel SNR) of multi-relay with perfect CSI is given by: $C_{FBL}^{\text{perf}} = \mathbb{E}_{\gamma} [C_{FBL,i}]$.

IV. AVERAGE CSI SCENARIO: IBL-THROUGHPUT VS. FBL-THROUGHPUT

A. The IBL-Throughput of Multi-Relay with Average CSI

Our previous work [5] has shown that the outage probability of a multi-relay transmission with average CSI is given by:

$$P_{MR}^{\text{out}} = \sum_{n=0}^J \mathcal{P}_{R,D}^{\text{out}}(n) \cdot \mathcal{P}_B(n; J, P_{S,R}^{\text{out}}), \quad (7)$$

where the number of active relays n is a binomially distributed random variable. Recall that J is the total number of relays deployed in the system. Denote by $\mathcal{P}_B(n; J, P_{S,R}^{\text{out}})$ the probability density function of n , hence we have:

$$\mathcal{P}_B(n; J, P_{S,R}^{\text{out}}) = \binom{J}{n} (1 - P_{S,R}^{\text{out}})^n (P_{S,R}^{\text{out}})^{J-n}, \quad (8)$$

where $P_{S,R}^{\text{out}}$ is the outage probability of the link from the source to each relay:

$$P_{S,R}^{\text{out}} = 1 - \exp(-\gamma^* \sigma^2 / 2\bar{h}_{S,j}^2 P_{\text{tx}}). \quad (9)$$

In (7), $\text{Pr}_2(n)$ is the outage probability of the relaying phase with n active relays. In addition, the SNR threshold γ^* is subject to the coding rate r and is given by $\gamma^* = 2^r - 1$. Due to MRC, the combined SNR at the destination results from the superposition of several fading signals, which leads to a Gamma-distributed random variable for the joint SNR [16]. Hence, $\mathcal{P}_{R,D}^{\text{out}}(n)$ is equivalent to the cumulative distribution function of a Gamma-distributed random variable:

$$\mathcal{P}_{R,D}^{\text{out}}(n) = \begin{cases} 1 - \sum_{j=0}^{n-1} \frac{1}{j!} \left(\frac{\gamma^*}{\beta}\right)^j e^{-\frac{\gamma^*}{\beta}}, & n > 0; \\ 1, & n = 0, \end{cases} \quad (10)$$

where β is the scaling parameter of the gamma distribution and is given by $\beta = 2 \sum_{j=1}^J P_{\text{tx}} \bar{h}_{j,D}^2 / J \sigma^2$. Both the gamma distribution and the binomial distribution are approximations based on the topology simplification (recall that the distances between relays are assumed to be significantly shorter than the distances either from the source to relays or from the relays to the destination, this simplification further assumes distances from the source to relays to be the same).

Finally, the throughput under the IBL regime with average CSI (also known as outage capacity) of the studied multi-relay system is given by: $C_{\text{IBL}}^{\text{ave}} = (1 - P_{\text{MR}}^{\text{out}})r/2$.

B. The FBL-Throughput of Multi-Relay with Average CSI

Based on the topology simplification [5], the error probabilities of the links from the source to relays can be simplified to be approximately the same. Hence, the expected error probability of a source-relay link is:

$$\begin{aligned} \mathbb{E}_{\gamma} [\varepsilon_{S,R}] &= \frac{\sigma^2}{P_{\text{tx}} |\bar{h}_2|^2} \int_0^{\infty} e^{-\frac{\gamma \sigma^2}{P_{\text{tx}} |\bar{h}_2|^2}} \mathcal{P}(\gamma, r, m) d\gamma \\ &= \frac{\sigma^2}{\sqrt{2\pi P_{\text{tx}} |\bar{h}_2|^2}} \int_0^{\infty} \int_{\sqrt{mw}(\gamma)}^{\infty} e^{-\frac{t^2 P_{\text{tx}} |\bar{h}_2|^2 + 2\gamma \sigma^2}{2P_{\text{tx}} |\bar{h}_2|^2}} dt d\gamma, \end{aligned} \quad (11)$$

where $w(\gamma) = \frac{\mathcal{C}(\gamma) - r}{\sqrt{\frac{1}{m}(1 - 2^{-2\mathcal{C}(\gamma)}) \log_2 e}}$.

Then, the number of relays which decode the data block successfully is binomial-distributed:

$$\mathcal{P}_B(n, N, \varepsilon_{S,R}) = \binom{N}{n} \left(1 - \mathbb{E}_{\gamma} [\varepsilon_{S,R}]\right)^n \mathbb{E}_{\gamma} [\varepsilon_{S,R}]^{N-n}, \quad (12)$$

Next, the received SNR at the destination is a Gamma-distributed random variable (the proof is the same as the one in the IBL regime, which is given in [16]). Hence, if the number of forwarding relays is n , the expected error probability of the second hop can be obtained by averaging (2) over the Gamma distribution. Denote this error probability by $\bar{\varepsilon}_{R,D}(n)$, the expected overall error probability (based on the topology simplification) is furthermore given by:

$$\mathbb{E}_{\gamma} [\varepsilon_{MR}] = \sum_{n=0}^N \mathbb{E}_{\gamma} [\varepsilon_{R,D}] \mathcal{P}_B(n, N, \mathbb{E}_{\gamma} [\varepsilon_{S,R}]). \quad (13)$$

Finally, the expected FBL-throughput is given by: $C_{\text{FBL}}^{\text{ave}} = (1 - \mathbb{E}_{\gamma} [\varepsilon_{MR}])r/2$.

V. NUMERICAL RESULTS

In this section we numerically compare the throughputs of multi-DF-relay under the FBL assumption and under the IBL assumption. For the comparison, we consider the following parameterization of the system model: First, we consider an outdoor urban scenario and distances of the broadcasting, relaying and direct links are set to 200 m, 200 m and 360 m. Second, we set $P_{\text{tx}} = 30$ dBm and

$\sigma^2 = -90$ dBm. Moreover, we utilize the well-known COST [17] model with the center frequency 2 GHz for calculating the path-loss.

A. Impact of Relay Number on the Relaying Performance

In Figure 2, we show the relationship between the relaying performance and the number of relays deployed in the system. Firstly, as more relays are deployed in the

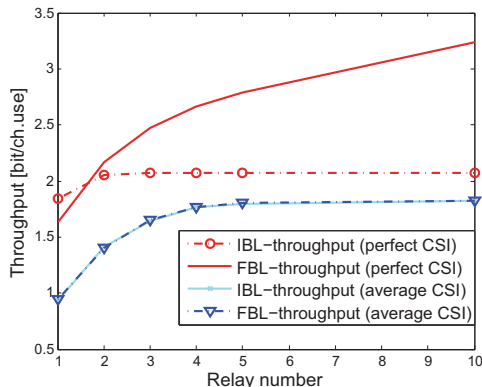


Fig. 2. IBL-throughput vs. FBL-throughput while varying the number of relays deployed in the system ($m=500$).

system both IBL-throughputs and FBL-throughputs are improved. Secondly, with the average CSI at the source multi-relay transmission has similar performance under the IBL and FBL regimes. This observation matches our previous results in [15] for a single relay scenario. Thirdly and most surprisingly, we find that with perfect CSI the FBL-throughput of multi-relay transmission exceeds the IBL-throughput if there are two or more relays deployed in the system. Recall that the IBL-throughput is modeled based on the Shannon capacity which is generally expected to be an upper limit of the channel's capacity/throughput. Hence, one could expect that the performance with IBL assumption should be the upper limit of the one under the FBL regime. However, our results show that under the studied multi-relay system the upper throughput limit is not the IBL-throughput anymore. This is surprising and different from our findings in [15], where we showed for a single relay system that the IBL-throughput is always higher than the FBL-throughput. An explanation is as follows. According to (2), if the coding rate of a single-link transmission is higher than the Shannon capacity of the link, the error probability of this transmission is higher than 0.5 but lower than 1. In other words, in the considered multi-relay system even if the source sets the coding rate to be higher than the Shannon capacity of each source-relay link, based on the FBL model it is possible that some relays decode the data block correctly. However, under the IBL regime there is definitely an error for the link which has a coding rate being higher than the Shannon capacity. As a result, under the FBL assumption the multi-relay system can achieve a higher throughput (in comparison to under the IBL

assumption) by setting the coding rate more aggressively for each transmission period (according to the perfect CSI). Our finding indicates that the Shannon capacity is inappropriate for modeling the capacity/throughput of the studied system where the transmission is not completely error-free, i.e., it is possible that the overall transmission is successful while only partial relays decode the packet correctly (from the source) and join the forwarding. Lastly, the mismatch between the FBL-throughput and IBL-throughput (under the perfect CSI scenario) is significantly increasing in the number of relays deployed in the system.

B. Impact of Blocklength on Relaying Performance

A condition for FBL-throughput violating the IBL-throughput is shown in Figure 2: The number of relays should be at least two. As the FBL-throughput is influenced by the blocklength, in this subsection we further investigate the impact of blocklength on this violation for the scenario with only two relays. We show it in Figure 3

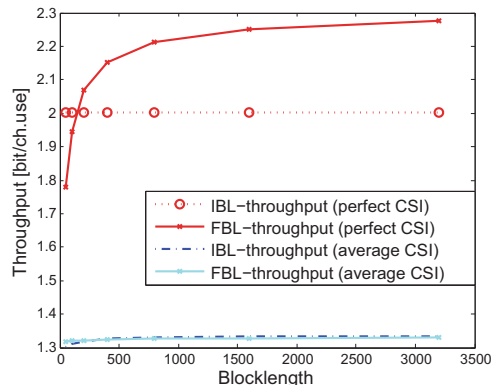


Fig. 3. The performance of two relays while varying the blocklength.

and find that the FBL-throughput is increasing in the blocklength while the IBL-throughput is not influenced by the blocklength. As a result, the FBL-throughput is lower than the IBL-throughput for very short blocklengths but significantly higher than the IBL-throughput when the block is long.

VI. CONCLUSION

In this work, we studied the performance of multi-DF-relay under the FBL regime. We found that with perfect CSI at the source the FBL-throughput is more likely higher than the IBL-throughput. It is known that for a single link transmission the IBL performance is the upper limit of the FBL performance. Our work actually provided an example to show that this result (of single link transmission) is not always true for more complex systems, e.g., the considered multi-DF-relay network. It should be mentioned that the information-theoretic performance limit of the multi-DF-relay network is still open and requires more future work.

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