Ultra-Reliable Low-Latency Communication of Periodic and Event-Triggered Dependable Traffic Streams

Antonios Pitarokoilis, James Gross, Mikael Skoglund
School of Electrical Engineering and Computer Science
KTH – Royal Institute of Technology, Stockholm, Sweden
apit@kth.se, {james.gross,skoglund}@ee.kth.se

***Abstract*—The efficient design of ultra-reliable low-latency communication (URLLC) is a major research objective for next generation wireless systems, in particular for industrial automation applications. Massive MIMO has been successful in providing high spectral and energy efficiency, and it is of importance to investigate the potential gains and limitations it exhibits when applied for URLLC. We study a scenario where two sets of nodes with different traffic characteristics communicate with a central node equipped with multiple antenna elements. We characterize the outage probability when fully orthogonal training sequences are used versus sharing of the training sequences between the two sets of nodes. It is shown that substantial performance gains can be reaped with shared training sequences when there are strict latency requirements and/or large number of nodes to be served.**

I. Introduction

The efforts of researchers and the industry to advance wireless communication systems have focused so far on providing higher data rates to the served users and improving the energy efficiency of the networks. However, the fifth generation (5G) of wireless networks is also expected to support massive connectivity of multiple Internet-of-Things (IoT) devices and ultra-reliable, low-latency communication (URLLC) connections [1]. These additional services will cause a major disruption in the design of 5G networks [2].

When the objective is to provide high data rates for applications with loose latency requirements or large amounts of data, such as web browsing or video streaming, reliable communication can be achieved by coding over large blocks of information, as dictated by Shannon's capacity results. Further, the additional control signaling that is required to establish a connection is not significant. In contrast, URLLC includes transmission of short packets, with strict latency and reliability requirements, which are sometimes triggered by random, external events such as alarms. The combination of low-latency, high reliability and potential intermittence of communication impose considerable challenges to the system design of URLLC [3].

The use of multiple antennas has undeniably transformed the design of wireless networks in the recent decades. Recently, Massive MIMO was shown to provide unprecedented gains in spectral and energy efficiency [4]. However, one can use the large number of base station antennas to achieve high reliability instead of aggressive spatial multiplexing. Hence, it is important to investigate the potential gains and the limitations of Massive MIMO in the area of URLLC. Due to channel hardening, Massive MIMO can effectively mitigate deep fades in the wireless channel and, consequently, can provide a reliable communication link. However, channel hardening can only be exploited when there is accurate knowledge of the propagation channel. This knowledge is typically acquired via dedicated training, which increases the latency and reduces the transmission rate. A rigorous characterization of this latency-reliability trade-off is necessary.

In this work, we consider an uplink scenario, where two sets of single-antenna nodes, termed *periodic* and *sporadic*, respectively, communicate over the same time and frequency resources with a central node equipped with a large number of antennas. Both periodic and sporadic nodes communicate blocks of critical information that must be delivered with strict latency requirements. A block of information arrives at each periodic node at constant time intervals, hence, the periodic nodes transmit at every transmission opportunity. In contrast, a block of information arrives at a sporadic node randomly, hence, a sporadic node transmits (is *active*) only at the transmission opportunities that a block of information has arrived. An example, where such a scenario can arise is the case of industrial automation, where a set of sensors (periodic nodes) communicates periodically its measurements towards an information fusion center (central node) and a set of safety nodes (sporadic nodes) communicate their alarm signals sporadically.

The central node acquires an estimate of the channel from each node by dedicated channel training. An accurate channel estimate for every node can be acquired only if each node is assigned an orthogonal training sequence. However, with strict latency constraints, large number of nodes and low probability that a sporadic node is active, such a training sequence assignment might be wasteful of resources. Orthogonal training requires longer training sequences, which might exhaust the latency constraint and reduce the available time for information transmission. A simple solution to reduce the training overhead significantly is to assign the same training sequence to a pair of one periodic node and one sporadic node. However, in this case, the central node acquires a single estimate for each pair of nodes, which adversely affects the system performance.

In this work, the latency-reliability trade-off of the two different training sequence assignments is characterized in terms of the outage probability under various operating conditions and decoding strategies. It is shown that considerable gains in terms of reliability can be achieved with non-orthogonal training sequence assignment by means of simple signal processing. Also, non-orthogonal assignment can achieve a more balanced performance between the two groups of nodes. Further, it is expected that better performance can be achieved by using more sophisticated receive strategies and by appropriately exploiting statistical information about the propagation channels.

Notation: x denotes a scalar constant, x a scalar random variable and x its realization. Vectors are denoted with bold lower case letters and matrices with bold upper case letters. The same convention with that of scalars, holds for vectors and matrices, to distinguish constants, random variables and random variable realizations. Also, for short we use $\mathcal{R}(x) = \log_2(1+x)$ and $\mathrm{x} \overset{d}{=} \mathrm{y}$ denotes that the random variable x is *equal in distribution* to the random variable y.

II. System Model and Channel Estimation

The received signal at the central node on the i-th channel use is given by

$$\mathbf{y}[i] = \sqrt{\rho_p}\sum_{k=1}^{K}\mathbf{h}_k\mathrm{x}_k[i] + \sqrt{\rho_s}\sum_{k=K+1}^{2K}\vartheta_k\mathbf{h}_k\mathrm{x}_k[i] + \mathbf{z}[i], \quad (1)$$

where $\rho_p,\ \rho_s$ are the power constraints for the periodic and the sporadic nodes during data transmission, respectively, $\mathbf{h}_k \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \boldsymbol{I}_M)$ is the channel vector of the k-th node and is independent of the channel vectors of the other nodes, $\mathrm{x}_k[i]$ is the transmitted symbol from the k-th node and $\vartheta_k \in \{0,1\},\ k = K+1,\ldots,2K$, is a binary variable set to 1 when the corresponding sporadic node is active, else set to 0. The additive white Gaussian noise vector is denoted by $\mathbf{z}[i] \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \boldsymbol{I}_M)$.

The channels remain constant for a coherence interval of N_c channel uses and change to an independent realization after that. However, a latency constraint of N_L channel uses is assumed such that $N_L < N_c$ and, hence, the transmission sees only one realization of the channel fading.

A. Estimation with Shared Training Sequences

Let $\boldsymbol{x}_k \in \mathbb{C}^{K\times 1}$ be the k-th training sequence, $k = 1,\ldots,K$, such that $\boldsymbol{x}_k^H\boldsymbol{x}_q = K\mathbb{I}\{k=q\}$, where $\mathbb{I}\{\cdot\}$ is the indicator function. The received signal at the central node during training is given by

$$\mathbf{Y}_{\text{tr}} = \sqrt{\varrho_p}\sum_{k=1}^{K}\mathbf{h}_k\boldsymbol{x}_k^H + \sqrt{\varrho_s}\sum_{k=K+1}^{2K}\mathbf{h}_k\boldsymbol{x}_{k-K}^H + \mathbf{Z}_{\text{tr}}, \quad (2)$$

where $\text{vec}(\mathbf{Z}_{\text{tr}}) \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \boldsymbol{I}_{KM})$ is the vectorized additive white Gaussian noise matrix and $\varrho_p,\ \varrho_s$ are the power constraints during training of the periodic and sporadic nodes, respectively. The central node correlates the received matrix $\mathbf{Y}_{\text{tr}}$ with the training sequence $\boldsymbol{x}_k$ and uses the statistic $\mathbf{y}_{\text{tr},k} = \frac{1}{\sqrt{K}}\mathbf{Y}_{\text{tr}}\boldsymbol{x}_k$ to calculate the MMSE estimates for the k-th periodic node and the $(k+K)$-th sporadic node, that constitute a pair of nodes that share the same training sequence. The MMSE estimate for the k-th periodic node is given by $\hat{\mathbf{h}}_k = \frac{\hat{c}_p}{\sqrt{\varrho_p K}}\mathbf{y}_{\text{tr},k} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \hat{c}_p\boldsymbol{I}_M),\ \hat{c}_p = \frac{\varrho_p K}{\varrho_p K + \varrho_s K + 1}$. Similarly, the MMSE estimate for the $(k+K)$-th sporadic node is given by $\hat{\mathbf{h}}_{k+K} = \frac{\hat{c}_s}{\sqrt{\varrho_s K}}\mathbf{y}_{\text{tr},k} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \hat{c}_s\boldsymbol{I}_M),\ \hat{c}_s = \frac{\varrho_s K}{\varrho_p K + \varrho_s K + 1}$. The associated estimation error vectors are denoted by $\tilde{\mathbf{h}}_k$. Observe that the two channel estimates are colinear, in particular, they are related as $\hat{\mathbf{h}}_{k+K} = \sqrt{\frac{\varrho_s}{\varrho_p}}\hat{\mathbf{h}}_k$, hence, the quality of estimation is compromised in order to be able to serve more nodes with strict latency requirements.

Remark 1: When all the nodes in the system are assigned orthogonal training sequences (in this case the length of the sequences is $2K$), the MMSE channel estimate for the k-th periodic node is $\hat{\mathbf{h}}_k^{\perp} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \hat{c}_p^{\perp}\boldsymbol{I}_M),\ \hat{c}_p^{\perp} = \frac{2\varrho_p K}{2\varrho_p K + 1}$. Similarly, for the $(k+K)$-th sporadic node, the MMSE channel estimate is given by $\hat{\mathbf{h}}_{k+K}^{\perp} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \hat{c}_s^{\perp}\boldsymbol{I}_M),\ \hat{c}_s^{\perp} = \frac{2\varrho_s K}{2\varrho_s K + 1}$.

III. Outage Probability

In this section we present the outage probabilities under various operating conditions and decoding strategies. The precise definition of outage probability within this work is given in Definition 1, which follows in Section III-A. The choice of the outage probability is natural within this context, since the transmission experiences only one realization of the channel fading, hence, the use of the ergodic capacity cannot be justified. Further, relating to the motivating example from industrial automation, industries design their operation so that they adhere to predetermined safety integrity levels. In our scenario, this translates to a maximum violation probability requirement for a latency-constrained operation[1]. Since the goal is to design the system according to those safety integrity levels, it is of fundamental importance to define the relevant measure of error and investigate its behavior.

The central node uses the acquired channel estimates, $\hat{\mathrm{H}} = \left\{\hat{\mathbf{h}}_k : k = 1,\ldots,2K\right\}$, to perform matched filtering to the received signal, $\mathbf{y}[i]$, given by (1). The detected symbol for the k-th node is given by $\hat{\mathrm{x}}_k[i] = \hat{\mathbf{h}}_k^H\mathbf{y}[i]$.

A. Periodic Nodes with Inactive Paired Sporadic Nodes

In this case the sporadic node $k+K$, which is paired with the periodic node k, is inactive, hence, $\vartheta_{k+K} = 0$. The acquired estimate, however, is still a superposition of the channels of the k and $k+K$ nodes. The detected symbol, $\hat{\mathrm{x}}_k[i]$, is

$$\hat{\mathrm{x}}_k[i] = \hat{\mathbf{h}}_k^H\mathbf{y}[i] = \sqrt{\rho_p}\left\|\hat{\mathbf{h}}_k\right\|^2\mathrm{x}_k[i] + \hat{\mathbf{h}}_k^H\tilde{\mathbf{z}}_k[i], \quad (3)$$

where $\tilde{\mathbf{z}}_k[i] = \sqrt{\rho_p}\tilde{\mathbf{h}}_k\mathrm{x}_k[i] + \sqrt{\rho_p}\sum_{q\neq k}^{K}\mathbf{h}_q\mathrm{x}_q[i] + \sqrt{\rho_s}\sum_{q=K+1}^{2K}\vartheta_q\mathbf{h}_q\mathrm{x}_q[i] + \mathbf{z}[i]$. Using standard arguments [5, Ch. 2, App. C], the mutual information $I\left(\mathrm{x}_k[i];\hat{\mathrm{x}}_k[i]\middle|\hat{\mathrm{H}}\right)$ for the k-th periodic node with inactive paired sporadic node can be lower-bounded by

[1]In this context, operation can be understood by a simple communication task, or more complex, typically concatenated, operational steps.

$$I\left(\mathrm{x}_k[i];\hat{\mathrm{x}}_k[i]\middle|\hat{\mathrm{H}}\right) \geq R_k(\hat{\mathrm{H}}) = \mathcal{R}\left(\frac{\rho_p\left\|\hat{\mathbf{h}}_k\right\|^4}{\hat{\mathbf{h}}_k^H\mathbf{C}_k\hat{\mathbf{h}}_k}\right), \quad (4)$$

where $\mathbf{C}_k = \rho_p\sum_{q\neq k}^{K}\beta_q\hat{\mathbf{h}}_q\hat{\mathbf{h}}_q^H + \xi_k\boldsymbol{I}_M$, $\beta_q = 1 + \vartheta_{q+K}\left(\frac{\rho_s}{\rho_p}\right)^2$ and $\xi_k = \rho_p K(1-\hat{c}_p)+\rho_s\left|\mathcal{A}_s\right|(1-\hat{c}_s)+1$. $\mathcal{A}_s$ is the set of active sporadic nodes and $|\mathcal{A}_s| = \sum_{q=K+1}^{2K}\vartheta_q$ is its cardinality.

Remark 2: For the case of fully orthogonal training, (4) is still a lower bound on the mutual information for the periodic and active sporadic nodes by simply setting $\hat{\mathbf{h}}_k \mapsto \hat{\mathbf{h}}_k^{\perp}$, $\hat{c}_p \mapsto \hat{c}_p^{\perp}$, $\hat{c}_s \mapsto \hat{c}_s^{\perp}$ and $\mathbf{C}_k = \rho_p\sum_{\substack{q=1\\q\neq k}}^{K}\hat{\mathbf{h}}_q^{\perp}\left(\hat{\mathbf{h}}_q^{\perp}\right)^H + \rho_s\sum_{\substack{q=K+1\\q\neq k}}^{2K}\vartheta_q\hat{\mathbf{h}}_q^{\perp}\left(\hat{\mathbf{h}}_q^{\perp}\right)^H + \xi_k^{\perp}\boldsymbol{I}_M$.

Definition 1: The k-th node is in outage when $R_k\left(\hat{\mathrm{H}}\right) < r_{\text{th}}$, where r_{th} is a threshold spectral efficiency. The outage probability is given by $p_{\text{out}}(r_{\text{th}}) = \mathbb{P}\left\{R_k\left(\hat{\mathrm{H}}\right) < r_{\text{th}}\right\}$. [2]

Proposition 1: For a selected transmission rate η_k, the outage probability for the k-th periodic node with inactive paired sporadic node is given by

$$p_{\text{out},p}(\eta_k) = \mathbb{P}\left\{\mathcal{R}\left(\frac{\rho_p\hat{c}_p\mathrm{u}_p}{\mathsf{IFPN}_k}\right) < \frac{\eta_k}{\varpi}\right\}, \quad (5)$$

where $\mathrm{u}_p \sim \chi^2_{2M}$, $\mathsf{IFPN}_k = \rho_p\hat{c}_p\left(1+\left(\frac{\rho_s}{\rho_p}\right)^2\right)\mathrm{w}_s + \rho_p\hat{c}_p\mathrm{w}_p + 2\xi_k$, $\mathrm{w}_s \sim \chi^2_{2|\mathcal{A}_s|}$, $\mathrm{w}_p \sim \chi^2_{2(K-|\mathcal{A}_s|-1)}$ and $\varpi = 1-\frac{K}{N_L}$.

Proof: For a fixed $\hat{\mathbf{h}}_k = \hat{\mathsf{h}}_k$ we study the probability $\mathbb{P}\left\{R_k < \frac{\eta_k}{\varpi}\middle|\hat{\mathbf{h}}_k = \hat{\mathsf{h}}_k\right\}$. Using Lemma 1, $\hat{\mathsf{h}}_k^H\mathbf{C}_k\hat{\mathsf{h}}_k \overset{d}{=} \left\|\hat{\mathsf{h}}_k\right\|^2\left(\rho_p\sum_{q\neq k}^{K}\frac{\hat{c}_p\beta_q}{2}\mathrm{v}_q + \xi_k\right)$, where $\mathrm{v}_q \sim \text{Exp}(2)$ is a sequence of i.i.d. exponential random variables with mean 2. By grouping the terms with $\vartheta_q = 1$ and $\vartheta_q = 0$ in two different groups and using the fact that $\sum_{i=1}^{n}\mathrm{v}_i \sim \chi^2_{2n}$, it holds $\frac{2}{\|\hat{\mathsf{h}}_k\|^2}\hat{\mathsf{h}}_k^H\mathbf{C}_k\hat{\mathsf{h}}_k \overset{d}{=} \mathsf{IFPN}_k$. Since $\frac{2}{\hat{c}_p}\left\|\hat{\mathbf{h}}_k\right\|^2 \overset{d}{=} \mathrm{u}_p \sim \chi^2_{2M}$ the result follows. ■

Corollary 1: For a selected transmission rate η_k when fully orthogonal training is applied, the outage probability for the k-th periodic node is given by

$$p^{\perp}_{\text{out},p}(\eta_k) = \mathbb{P}\left\{\mathcal{R}\left(\frac{\rho_p\hat{c}_p^{\perp}\mathrm{u}_p}{\rho_p\hat{c}_p^{\perp}\mathrm{w}_p+\rho_s\hat{c}_s^{\perp}\mathrm{w}_s+2\xi_k^{\perp}}\right) < \frac{\eta_k}{\varpi^{\perp}}\right\}, \quad (6)$$

where $\mathrm{u}_p \sim \chi^2_{2M}$, $\mathrm{w}_p \sim \chi^2_{2(K-1)}$, $\mathrm{w}_s \sim \chi^2_{2|\mathcal{A}_s|}$ and $\varpi^{\perp} = 1-\frac{2K}{N_L}$. The respective outage probability for the k-th active sporadic node is given by

$$p^{\perp}_{\text{out},s}(\eta_k) = \mathbb{P}\left\{\mathcal{R}\left(\frac{\rho_s\hat{c}_s^{\perp}\mathrm{u}_s}{\rho_p\hat{c}_p^{\perp}\mathrm{w}_p+\rho_s\hat{c}_s^{\perp}\mathrm{w}_s+2\xi_k^{\perp}}\right) < \frac{\eta_k}{\varpi^{\perp}}\right\}, \quad (7)$$

where $\mathrm{u}_s \sim \chi^2_{2M}$, $\mathrm{w}_p \sim \chi^2_{2K}$ and $\mathrm{w}_s \sim \chi^2_{2(|\mathcal{A}_s|-1)}$.

[2]The outage event is defined in this work with respect to a lower bound on the mutual information (see (4)). Hence, the derived outage probability is an upper bound on the true outage probability.

B. Periodic Nodes with Active Paired Sporadic Nodes

In this case, matched filtering results in a corresponding 2-node multiple access channel (MAC). We study the outage performance of successive interference cancellation (SIC). In particular, we study the outage performance under both possible decoding orders of SIC, i.e., decoding either first the periodic node (*P-S*) or first the sporadic node (*S-P*).

Proposition 2: The mutual information $I\left(\mathrm{x}_k[i];\hat{\mathrm{x}}_k[i]\middle|\hat{\mathrm{H}}\right)$ for the k-th periodic node, when the paired sporadic node is active and the decoding order is *P-S*, is lower-bounded by (4) where $\mathbf{C}_k = \rho_p\left(\frac{\rho_s}{\rho_p}\right)^2\hat{\mathbf{h}}_k\hat{\mathbf{h}}_k^H + \rho_p\sum_{q\neq k}^{K}\beta_q\hat{\mathbf{h}}_q\hat{\mathbf{h}}_q^H + \xi_k\boldsymbol{I}_M$. The corresponding lower bound for the paired active sporadic node, $k+K$, is $I\left(\mathrm{x}_{k+K}[i];\hat{\mathrm{x}}_{k+K}[i]\middle|\hat{\mathrm{H}}\right) \geq R_{k+K}(\hat{\mathrm{H}})$ where

$$R_{k+K}(\hat{\mathrm{H}}) = \mathcal{R}\left(\frac{\rho_s\left\|\hat{\mathbf{h}}_{k+K}\right\|^4}{\hat{\mathbf{h}}_{k+K}^H\mathbf{C}_{k+K}\hat{\mathbf{h}}_{k+K}}\right). \quad (8)$$

The noise covariance matrix is given by

$$\mathbf{C}_{k+K} = \rho_p\hat{\mathbf{h}}_k\hat{\mathbf{h}}_k^H\varsigma_k + \rho_p\sum_{q\neq k}^{K}\beta_q\hat{\mathbf{h}}_q\hat{\mathbf{h}}_q^H + \xi_k\boldsymbol{I}_M. \quad (9)$$

The binary variable ς_k is set to 0 when the decoding of the periodic node has been successful and to 1 when the decoding of the periodic node has been unsuccessful.

Proposition 3: For a selected pair of transmission rates (η_k, η_{k+K}), the outage probability for the k-th periodic node with active paired sporadic node when the decoding order is *P-S* is given by

$$p^{P\text{-}S}_{\text{out},p}(\eta_k) = \mathbb{P}\left\{\mathcal{R}\left(\mathsf{SINR}_k^{P\text{-}S}\right) < \frac{\eta_k}{\varpi}\right\}, \quad (10)$$

where

$$\mathsf{SINR}_k^{P\text{-}S} = \frac{\rho_p\hat{c}_p\mathrm{u}_p}{\rho_p\left(\frac{\rho_s}{\rho_p}\right)^2\hat{c}_p\mathrm{u}_p + \mathsf{IFPN}_k^{P\text{-}S}},$$

$\mathrm{u}_p \sim \chi^2_{2M}$ and $\mathsf{IFPN}_k^{P\text{-}S} = \rho_p\hat{c}_p\left(1+\left(\frac{\rho_s}{\rho_p}\right)^2\right)\mathrm{w}_s + \rho_p\hat{c}_p\mathrm{w}_p + 2\xi_k$, $\mathrm{w}_s \sim \chi^2_{2(|\mathcal{A}_s|-1)}$ and $\mathrm{w}_p \sim \chi^2_{2(K-|\mathcal{A}_s|)}$. The outage probability for the $(k+K)$-th active sporadic node is

$$\begin{aligned}p^{P\text{-}S}_{\text{out},s}(\eta_{k+K}) &= p^{P\text{-}S}_{\text{out},p}(\eta_k)\mathbb{P}\left\{\mathcal{R}\left(\mathsf{SINR}^{P\text{-}S}_{k+K}(1)\right) < \frac{\eta_{k+K}}{\varpi}\right\}\\ &+\left(1-p^{P\text{-}S}_{\text{out},p}(\eta_k)\right)\mathbb{P}\left\{\mathcal{R}\left(\mathsf{SINR}^{P\text{-}S}_{k+K}(0)\right) < \frac{\eta_{k+K}}{\varpi}\right\} \quad (11)\end{aligned}$$

where

$$\mathsf{SINR}^{P\text{-}S}_{k+K}(\varsigma_k) = \frac{\rho_s\hat{c}_s\mathrm{u}_s}{\rho_s\left(\frac{\rho_p}{\rho_s}\right)^2\hat{c}_s\mathrm{u}_s\varsigma_k + \mathsf{IFPN}^{P\text{-}S}_{k+K}},$$

$\mathsf{IFPN}^{P\text{-}S}_{k+K} = \rho_s\hat{c}_s\left(1+\left(\frac{\rho_p}{\rho_s}\right)^2\right)\mathrm{w}_s + \rho_s\hat{c}_s\left(\frac{\rho_p}{\rho_s}\right)^2\mathrm{w}_p + 2\xi_k$, $\mathrm{u}_s \sim \chi^2_{2M}$, $\mathrm{w}_s \sim \chi^2_{2(|\mathcal{A}_s|-1)}$ and $\mathrm{w}_p \sim \chi^2_{2(K-|\mathcal{A}_s|)}$.

Proposition 4: The mutual information $I\left(\mathrm{x}_{k+K}[i];\hat{\mathrm{x}}_{k+K}[i]\middle|\hat{\mathrm{H}}\right)$ for the $(k+K)$-th active sporadic node, when the decoding order is *S-P*, is lower-bounded by (8) and (9) for $\varsigma_k = 1$. The corresponding lower bound for the paired periodic node, k, are given by (4) where

$$\mathbf{C}_k = \rho_s\left(\frac{\rho_p}{\rho_s}\right)^2\hat{\mathbf{h}}_k\hat{\mathbf{h}}_k^H\varsigma_k + \rho_p\sum_{q\neq k}^{K}\beta_q\hat{\mathbf{h}}_q\hat{\mathbf{h}}_q^H + \xi_k\boldsymbol{I}_M, \quad (12)$$

where, similarly, ς_k is set to 0 when the decoding of the active sporadic node has been successful and to 1 else.

Proposition 5: For a selected pair of transmission rates (η_k, η_{k+K}), the outage probability for the $(k+K)$-th active sporadic node when the decoding order is *S-P* is given by

$$p^{\textit{S-P}}_{\text{out},s}(\eta_{k+K}) = \mathbb{P}\left\{\mathcal{R}\left(\mathsf{SINR}^{\textit{S-P}}_{k+K}\right) < \frac{\eta_{k+K}}{\varpi}\right\}, \tag{13}$$

where

$$\mathsf{SINR}^{\textit{S-P}}_{k+K} = \frac{\rho_s \hat{c}_s \mathrm{u}_s}{\rho_s \left(\frac{\rho_p}{\rho_s}\right)^2 \hat{c}_s \mathrm{u}_s + \mathsf{IFPN}^{\textit{S-P}}_{k+K}},$$

$\mathrm{u}_s \sim \chi^2_{2M}$ and $\mathsf{IFPN}^{\textit{S-P}}_{k+K} \overset{d}{=} \mathsf{IFPN}^{\textit{P-S}}_{k+K}$ in Proposition 3. The outage probability for the k-th periodic node is given by

$$p^{\textit{S-P}}_{\text{out},p}(\eta_k) = \left(1 - p^{\textit{S-P}}_{\text{out},s}(\eta_{k+K})\right) \mathbb{P}\left\{\mathcal{R}\left(\mathsf{SINR}^{\textit{S-P}}_{k}(0)\right) < \frac{\eta_k}{\varpi}\right\} + p^{\textit{S-P}}_{\text{out},s}(\eta_{k+K}) \mathbb{P}\left\{\mathcal{R}\left(\mathsf{SINR}^{\textit{S-P}}_{k}(1)\right) < \frac{\eta_k}{\varpi}\right\} \tag{14}$$

where

$$\mathsf{SINR}^{\textit{S-P}}_{k}(\varsigma_k) = \frac{\rho_p \hat{c}_p \mathrm{u}_p}{\rho_p \left(\frac{\rho_s}{\rho_p}\right)^2 \hat{c}_p \mathrm{u}_p \varsigma_k + \mathsf{IFPN}^{\textit{S-P}}_{k}},$$

$\mathrm{u}_p \sim \chi^2_{2M}$ and $\mathsf{IFPN}^{\textit{S-P}}_{k} \overset{d}{=} \mathsf{IFPN}^{\textit{P-S}}_{k}$ in Proposition 3.

IV. Numerical Examples

In this section we present numerical examples that reveal the performance trade-off between the different communication strategies. The values assigned to the parameters are summarized: $K = 30$, $|\mathcal{A}_s| = 15$, $M = 60$, $N_L = 100$, $\varrho_p = \rho_p$, $\varrho_s = \rho_s = 0$ dB. Two different scenarios are investigated, namely, when $\rho_s = \frac{\rho_p}{4}$ (strong periodic node) and when $\rho_s = 4\rho_p$ (strong sporadic node). In the strong periodic node scenario, the selected operating point (in bits per channel use (bpcu)) is $(\eta_k, \eta_{k+K}) = (0.4, 0.05)$, $\forall k = 1, \ldots, K$. In the strong sporadic node scenario the selected operating point is $(\eta_k, \eta_{k+K}) = (0.08, 0.5)$, $\forall k = 1, \ldots, K$. As a reference, in Table 1 the corresponding ergodic rates are tabulated. The selected operating points are in the corresponding ergodic rate regions for $\rho_p = 20$ dB and are chosen to be relatively close to the boundary. Hence, they are reasonable choices to balance outage probability and information rate.

In the Figs. 1 and 2, the curves labeled "Orthogonal Training" correspond to (6) and (7) in Corollary 1, the "Inactive Sporadic" correspond to (5) in Proposition 1, the "Active (P-S)" correspond to (10) and (11) Proposition 3 and the "Active (S-P)" to (13) and (14) in Proposition 5.

In Fig. 1 the outage probability of a periodic node is plotted as a function of ρ_p[3] in [dB] (Fig. 1a), as a function of the antennas M at the central node (Fig. 1b) and as a function of the latency constraint N_L (Fig. 1c). In all cases, substantial performance improvement is observed with respect to the case of fully orthogonal training sequences. In Fig. 1a the outage probability reaches a saturation level at high ρ_p. This is attributed to the matched filtering at the receiver, which is a suboptimal strategy against strong interference. In Fig. 1b the outage probability decays rapidly with the number of antennas, M, which verifies that even with rudimentary signal processing, the use of excess antennas can substantially improve the system performance. In Fig. 1c the performance gap diminishes as N_L grows large, since the training overhead becomes less significant. In all plots, the constant gap between the "Inactive Sporadic" and the "Active Sporadic (P-S)" is attributed to the additional interference from the active paired sporadic node. Finally, in Fig. 2, the strong sporadic nodes scenario is plotted. Similar behavior to that in Fig. 1 is observed.

[3]Due to the normalization of the channel statistics and the noise variance, ρ_p is proportional to the SNR.

Table 1: Ergodic Rates

	Strong Periodic		Strong Sporadic	
	Periodic	Sporadic	Periodic	Sporadic
Orthogonal	0.4198	0.1505	0.2099	0.5375
Inactive S	0.6325		0.0893	
P-S	0.6329	0.0620		
S-P			0.0925	0.7591

V. Conclusions

Substantial gains in outage probability can be obtained by sharing training sequences in comparison to the fully orthogonal training assignment, when the number of nodes is large and/or the latency requirements are very stringent. However, these gains cannot be obtained for both sporadic and periodic nodes simultaneously. The current study depicts the worst case propagation environment given that all the propagation channels have the same second order statistics. In environments that exhibit more correlation, the exploitation of this knowledge in node pairing and in signal processing is expected to yield better results, making the non-orthogonal assignment of pilots competitive in more general scenarios.

Appendix

Lemma 1: Let $\boldsymbol{v} \in \mathbb{C}^{M\times 1}$ a constant vector and $\mathbf{w} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, c\boldsymbol{I}_M)$. Then $\frac{2}{c\|\boldsymbol{v}\|^2}\left|\boldsymbol{v}^H\mathbf{w}\right|^2 \sim \chi^2_2$.

Proof: Define the unitary matrix $\boldsymbol{U} \in \mathbb{C}^{M\times M}$ such that the first column $\boldsymbol{u}_1 = \frac{\boldsymbol{v}}{\|\boldsymbol{v}\|}$ and $\boldsymbol{v}^H\boldsymbol{u}_i = 0$, $i = 2, \ldots, M$. It holds $\left|\boldsymbol{v}^H\mathbf{w}\right|^2 = \left|\boldsymbol{v}^H\boldsymbol{U}\boldsymbol{U}^H\mathbf{w}\right|^2 = \left|\|\boldsymbol{v}\|\,\boldsymbol{e}_1^T\boldsymbol{U}^H\mathbf{w}\right|^2 = \|\boldsymbol{v}\|^2\left|\boldsymbol{e}_1^T\boldsymbol{U}^H\mathbf{w}\right|^2$. Since $\boldsymbol{U}^H\mathbf{w} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, c\boldsymbol{I}_M) \Rightarrow \boldsymbol{e}_1^T\boldsymbol{U}^H\mathbf{w} \sim \mathcal{N}_{\mathbb{C}}(0, c)$ the result follows. ■

References

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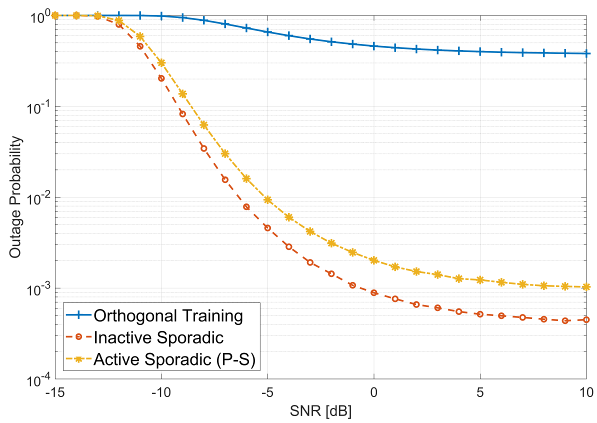
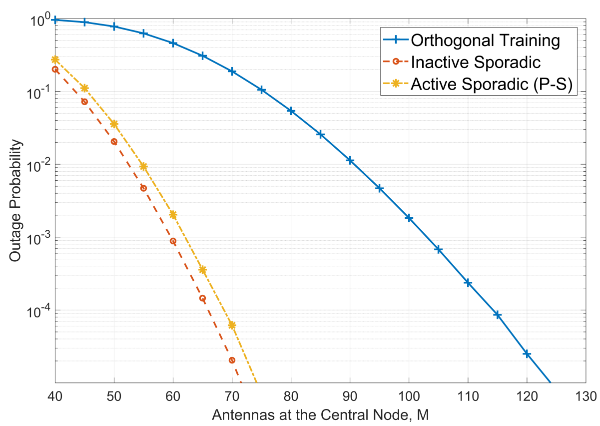
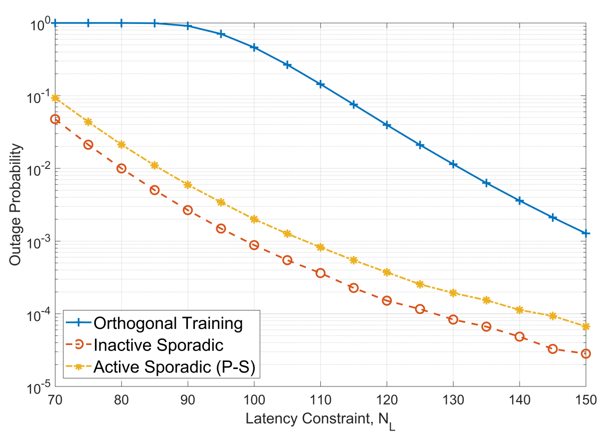


(a) Outage probability versus ρ_p, [dB].

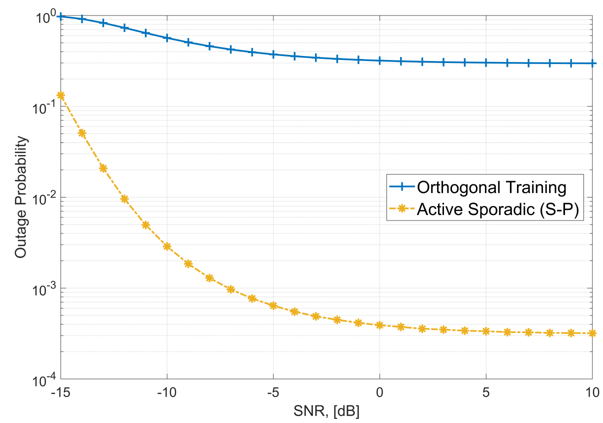


(b) Outage probability versus central node antennas, M.

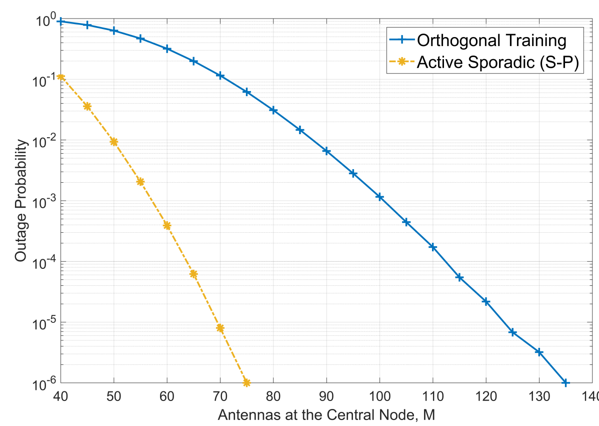


(c) Outage probability versus latency constraint, N_L.

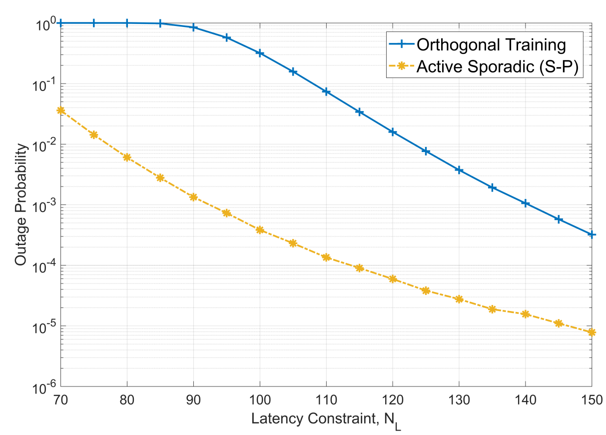
Fig. 1: Outage probability of periodic node when it is stronger than the sporadic node at the operating point $(\eta_k, \eta_{k+K}) = (0.4, 0.05),\ \forall k = 1, \dots, K$



(a) Outage probability versus ρ_p, [dB].



(b) Outage probability versus central node antennas, M.



(c) Outage probability versus latency constraint, N_L.

Fig. 2: Outage probability of sporadic node when it is stronger than the periodic node at the operating point $(\eta_k, \eta_{k+K}) = (0.08, 0.5),\ \forall k = 1, \dots, K$