Optimal Scheduling of Reliability-Constrained Relaying System under Outdated CSI in the Finite Blocklength Regime

Yulin Hu *IEEE Member*, Anke Schmeink, *IEEE Member* and James Gross, *IEEE Senior Member*

***Abstract*—Under the assumption of outdated channel state information (CSI) at the source, we maximize the finite blocklength (FBL) throughput of a two-hop relaying system while guaranteeing a reliability constraint We investigate the trade-off between the choice of so called scheduling weights to avoid transmission errors and the resulting coding rate. We show that the corresponding maximization of the throughput can be solved efficiently by iterative algorithms which require a recomputation of the scheduling weights prior to each transmission. Thus, we also study heuristics relying on choosing the scheduling weights only once. Through numerical analysis, we first provide insights on the structure of the throughout under different scheduling weights and channel correlation coefficients. We then turn to the comparison of the optimal scheduling with the heuristic and show that the performance gap between them is only significant for relay systems with high average signal-to-noise ratios (SNR) on the backhaul and relaying link. In particular, the optimal scheduling scheme provides most value in case that the data transmission is subject to strict reliability constraints, justifying the significant additional computational burden.**

***Index Terms*—decode-and-forward, finite blocklength, optimal scheduling, outdated CSI, relaying.**

I. Introduction

In wireless communications, relaying [1]–[3] is well known as an efficient way to mitigate fading by exploiting spatial diversity and providing better channel quality. Specifically, two-hop decode-and-forward (DF) relaying protocols significantly improve the coverage performance, throughput and quality of service [4]–[7]. However, typically these studies on the advantage of relaying are under the ideal assumption of communicating arbitrarily reliable at Shannon's channel capacity, i.e., code words are assumed to be infinitely long.

In the finite blocklength (FBL) regime, the data transmission is no longer arbitrarily reliable. Especially when the blocklength is short, the error probability (due to noise) becomes significant even if the rate is selected below the Shannon limit. Taking this into account, an accurate approximation of the achievable coding rate under the FBL assumption for an additive white Gaussian noise (AWGN) channel was derived in [8] for a single-hop transmission system. Subsequently, the initial work for AWGN channels was extended to Gilbert-Elliott channels [9], quasi-static fading channels [10], [11], quasi-static fading channels with retransmissions [12], spectrum sharing networks [13], transmissions with packet scheduling [14] as well as networks with energy harvesting nodes [15], [16] and operating under network coding [17]. It is shown in these works that the FBL performance of a single-hop transmission is determined by the coding rate, error probability and blocklength. In particular, the performance loss due to the additional decoding errors at FBL is considerable and increases as the blocklength decreases. Also, if the channel and the blocklength are given, the error probability of the single-hop transmission is strictly increasing in the coding rate. In our own previous work [18]–[20], we extended Polyanskiy's model [8] of single-hop transmission to a two-hop DF relaying network, where the relay halves the distance to provide a power gain but at the same time also halves the blocklength of the transmission. Subsequently, we provided a general analytical model of the FBL performance under static/quasi-static channels in [18]–[21] while assuming the transmitter to have only average CSI. More recently, the reliability and the link-layer performance [22], [23] of a relaying network with FBL codes was studied under the perfect CSI assumption.

In practical relay systems (as for instance specified by the LTE standard) CSI feedback mechanisms are usually implemented, i.e., allowing the receiver to instantaneously estimate and feedback the CSI to the transmitter. However, typically there exists a delay between the instant of sampling the channel and the point in time when this CSI sample is received by the transmitter making the CSI feedback delayed and therefore outdated. The performance analysis and optimization of relaying systems operating on outdated CSI have been widely discussed in the infinite blocklength (IBL) regime. In [24], the probability of an outage event (defined as the event when the coding rate is higher than the Shannon capacity) of a DF relaying network is studied under the outdated CSI relaying scenario. Protocols are designed in [25] for a relay system operating based on outdated CSI to optimally trade-off outage, delay, and throughput. For multi-relay scenarios with outdated CSI, optimal relay selection algorithms [26], [27] are proposed to minimize the outage probability. However, these works generally ignore the impact of transmitting under FBL restrictions, which introduces further subtleties in addition to the imperfect channel knowledge.

In this paper, we thus study the FBL performance of a relaying network assuming the source to have only outdated CSI, i.e., the CSI is delayed and inaccurate. Hence, the reliability performance of the network is influenced by both the FBL impact and the CSI accuracy. In particular, we consider a more general case where the delays of CSI of the two links of relaying are different, which results in the inaccuracies of the outdated CSI being different for the two hops of relaying. In addition, the transmissions are assumed to satisfy certain reliability constraints. Note that different from the average CSI scenario considered in [18]–[20], based on the provided CSI the source is able to adjust the coding rate per frame. The major contribution of this work is to answer the question how to optimally schedule the coding rate per frame based on the outdated CSIs while guaranteeing the transmission reliability. Moreover, as objective function we focus on the maximization of the FBL throughput. To solve the problem, we propose a pessimistic approach to guarantee the reliability. In particular, we propose to let the source choose the coding rate based on scheduling weights, i.e. factors by which the outdated channel SNRs are rescaled. The detailed contributions of this work are as follows:

- We first derive a model for the FBL throughput of the relaying system operating based on outdated CSI.
- Next, we propose an optimal scheduling scheme that maximizes the FBL throughput. We show that the objective function of the scheduling problem is concave in the coding rate and quasi-concave in the scheduling weights. Therefore, the optimal scheduling problem can be solved efficiently by iterative methods.
- Nevertheless, to mitigate the computational complexity, we also consider a sub-optimal scheduling scheme, where fixed scheduling weights are applied per frame. We refer to this scheme as constant heuristic and study the problem of choosing the constant scheduling weights which maximize the average FBL throughput over time.
- We finally perform numerical evaluations and show that the optimal scheme outperforms the best constant heuristic, especially when the reliability constraint is strict and/or the average SNR is high. Surprisingly, we find that the channel correlation has only a marginal impact on the performance gap between the two schemes.

The rest of the paper is organized as follows. Section II describes the system model and briefly reviews the background regarding the FBL regime. In Section III, we first derive the FBL performance model of the considered relaying scenario with outdated CSI. Afterwords, we state the optimization problem of interest and provide the theoretical insights that lead to the optimal solution. We then turn to the constant heuristic and provide an optimal solution for choosing the fixed scheduling weight. In Section IV we then present our numerical results. Finally, we conclude our work in Section V.

II. System Model

We consider a straightforward scenario with a source S, a destination D and a relay R as schematically shown in Figure 1. The relay is assumed to work under a DF principle.

Fig. 1. Example of the considered DF relaying scenario.

The entire system operates in a slotted fashion where time is divided into frames of length $n+2m$ symbols, as shown in Figure 2. Each frame consists of two parts, the initialization part and the transmission part. For the initialization part a

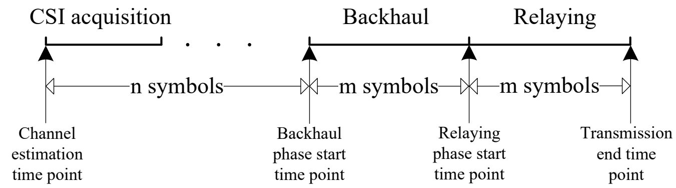

Fig. 2. Illustration of the relationship of channel estimation and data transmission within the considered frame time.

certain amount of symbols are spent, e.g., for a CSI acquisition/feedback period, a beacon period which contains the transmission schedule and a possible guard period (i.e., to ensure all transceivers to be standby). We assume this part to have a duration of n symbols, without specifying closer the exact system operation. The second part of each frame is the transmission part containing two phases, which are the backhaul phase (of length m) and the relaying phase (of length m). During the backhaul phase, the source sends a data block to the relay. Then, if the relay decodes the block successfully, it forwards the block to the destination in the subsequent relaying phase. Overall, we assume a setting where the initialization part takes a significantly longer amount of time than a single data transmission phase, i.e. $n \gg m$.

Channels are assumed to experience a time-varying Rayleigh-distributed random fading. As both the backhaul phase and the relaying phase are short, we assume that the channel state is constant during each phase. However, the channel states in different frames are assumed to be independent. Considering a frame i, the channel's complex states of the backhaul link and the relaying link are denoted by $h_{1,i}$ and $h_{2,i}$ and are assumed to be independent and identically distributed (i.i.d.). The received SNR at the relay of the backhaul phase and the received SNR at the destination of the relaying phase are denoted by $\gamma_{1,i}$ and $\gamma_{2,i}$. Hence, we have $\gamma_{k,i} = \bar{\gamma}_k h_{k,i}^2, k = 1,2$, where $\bar{\gamma}_k$ is the average SNR of link k (either the backhaul link or the relaying link). Recall that we assume the source to acquire the instantaneous CSI by sampling the channel n symbols prior to the backhaul phase and $n+m$ symbols prior to the relaying phase. Thus, due to the time-varying nature of the fading, the sampled channel coefficients, denoted by $\hat{h}_{k,i}, k = 1,2$, differ from the actual instantaneous channel coefficients $h_{k,i}$ that the data packet will experience. We adopt the widely-used Jakes model for the relation between $\hat{h}_{k,i}$ and $h_{k,i}$ [28], [29]:

$$h_{k,i} = \rho_k \hat{h}_{k,i} + \sqrt{1-\rho_k{}^2}\, e_{k,i}, \tag{1}$$

where $e_{k,i}$ is a complex Gaussian random variable, i.e., $e_{k,i} \sim \mathcal{CN}(0,1)$. In addition, $\rho_k, k = 1,2$ are channel correlation coefficients. Taking the frame sequence into account, we thus obtain $\rho_1 = J_0(2\pi f_{\mathrm{S-R}} n)$ and $\rho_2 = J_0(2\pi f_{\mathrm{R-D}}(n+m))$, where $f_{\mathrm{S-R}}$ and $f_{\mathrm{R-D}}$ stand for the Doppler frequency experienced on the backhaul link and the relaying link. In addition, $J_0(\cdot)$ denotes the zero-order Bessel function of the first kind [30]. Based on the outdated CSI $\hat{h}_{k,i}$, the outdated SNRs are given by $\hat{\gamma}_{k,i} = \bar{\gamma}_k \hat{h}_{k,i}^2, k = 1,2$. Thus, the instantaneous channel SNRs $\gamma_{k,i}$ become now random variables conditioned on the outdated SNRs $\hat{\gamma}_{k,i}$. The conditional probability density function (PDF) of the instantaneous SNRs of link k during frame i thus results to [29]:

$$\mathbb{P}\left[\gamma_{k,i}|\hat{\gamma}_{k,i}\right] = \frac{\exp(-\frac{\gamma_{k,i}+\rho_k^2\hat{\gamma}_{k,i}}{\bar{\gamma}_k(1-\rho_k^2)})}{\bar{\gamma}_k(1-\rho_k^2)} \cdot I_0\left(\frac{2\rho_k\sqrt{\gamma_{k,i}\hat{\gamma}_{k,i}}}{\bar{\gamma}_k(1-\rho_k^2)}\right), \quad (2)$$

where I_0 is the zero-order modified Bessel function of the first kind. We denote by $\bar{\bar{\gamma}}_{k,i}$ the median of the instantaneous SNR $\gamma_{k,i}$, for which the following equation holds:

$$\int_0^{\bar{\bar{\gamma}}_{k,i}} \mathbb{P}\left[\gamma_{k,i}|\hat{\gamma}_{k,i}\right] d\gamma_{k,i} = \int_{\bar{\bar{\gamma}}_{k,i}}^{+\infty} \mathbb{P}\left[\gamma_{k,i}|\hat{\gamma}_{k,i}\right] d\gamma_{k,i} = 0.5 \ . \quad (3)$$

Due to (1) the median of the distribution of the instantaneous channel $h_{k,i}$ is $\rho_k\hat{h}_{k,i}$, thus we have $\bar{\bar{\gamma}}_{k,i} \approx \rho_k^2\gamma_{k,i}$.

A. Finite Blocklength Error Model under Perfect CSI

For the additive white Gaussian noise (AWGN) channel [8] derives an accurate approximation of FBL performance for a single-hop transmission system. Subsequently, the result of the AWGN channel has been extended to a quasi-static fading channel mode [10], [11]: With a received SNR γ, blocklength m, block error probability ε, the coding rate (in bits per channel use) in a frame of a single-hop transmission is:

$$r = \mathcal{R}(\gamma, \varepsilon, m) \approx \mathcal{C}(\gamma) - \sqrt{\frac{V}{m}} Q^{-1}\left(\varepsilon\right), \quad (4)$$

where $\mathcal{C}\left(\gamma\right)$ is the Shannon capacity function in received SNR γ: $\mathcal{C}(\gamma) = \log_2\left(1+\gamma\right)$. In addition, $Q^{-1}(\cdot)$ is the inverse of the Q-function given by $Q\left(w\right) = \int_w^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$. Moreover, V is so-called the channel dispersion: $V = \left(1 - \frac{1}{(1+\gamma)^2}\right)(\log_2 e)^2$.

Then, for a single-hop transmission under a quasi-static fading channel, with blocklength m and coding rate r, the decoding (block) error probability at the receiver is given by:

$$\varepsilon = \mathcal{P}(\gamma, r, m) \approx Q\Big(\frac{\mathcal{C}(\gamma) - r}{\sqrt{V/m}}\Big). \quad (5)$$

Considering the channel fading, the expected/average error probability is given by [10]:

$$\mathop{\mathbb{E}}_{\gamma}\left[\varepsilon\right] = \mathop{\mathbb{E}}_{\gamma}\left[\mathcal{P}(\gamma, r, m)\right] \approx \mathop{\mathbb{E}}_{\gamma}\left[Q\Big(\frac{\mathcal{C}(\gamma) - r}{\sqrt{V/m}}\Big)\right]. \quad (6)$$

In the remainder of the paper, we investigate the considered relaying system in the FBL regime by applying the above approximations. As these approximations have been shown to be accurate for a sufficiently large blocklength m [8], for simplicity we will assume them to hold in equality in our analysis and numerical evaluation conditioned on the assumption of a sufficiently large value of m at each hop.

III. Maximizing the FBL Throughput under Reliability Constraints

As discussed in the previous section, the source has outdated CSI that it can rely on for scheduling the data transmission in the relay system. In this section, we address thus the problem of how to optimally schedule the coding rate based on the inaccurate outdated CSI such that the throughput of the relay system is maximized. We restrict this scheduling problem to a reliability constraint such that for each data transmission a target error probability ε_{th} must be met. Such a scheduling problem is justified by current discussions around industrial wireless communication systems, where small payload packets need to be transmitted within a bounded time interval while keeping a (stochastic) reliability guarantee. In the following, we first develop a throughput model of the relaying system with respect to the FBL assumption, building on Section II-A. Subsequently, the mathematical statement of the optimization problem is provided. We then turn to the solution, providing both an optimal solution as well as a low-complexity heuristic.

A. FBL Throughput Model for Relay Systems

Assuming r_i is the scheduled coding rate[1] for frame i with instantaneous SNRs $\gamma_{1,i}$ and $\gamma_{2,i}$, the overall error probability of the relaying system during frame i is:

$$\varepsilon_{\text{R},i}(r_i) = \varepsilon_{1,i} + \varepsilon_{2,i} - \varepsilon_{1,i}\varepsilon_{2,i}, \quad (7)$$

where $\varepsilon_{k,i} = \mathcal{P}(\gamma_{k,i}, r_i, m), i = 1, 2$. Based on (7), we immediately have the expected overall error probability conditioned on the outdated CSI $\hat{\gamma}$. It is the expected value of (7) over the conditioned channel fading distribution:

$$\bar{\varepsilon}_{\text{R},i}(r_i) = \bar{\varepsilon}_{1,i} + \bar{\varepsilon}_{2,i} - \bar{\varepsilon}_{1,i}\bar{\varepsilon}_{2,i}. \quad (8)$$

In (8), $\bar{\varepsilon}_{k,i},\ k = 1, 2$ are the expected error probabilities of either the backhaul link or the relaying link. Then, by averaging $\varepsilon_{k,i}$ over the conditional PDF in (2), $\bar{\varepsilon}_{k,i}, k = 1, 2$ is given by (9), where $\alpha(x, r_i) = \frac{\mathcal{C}(x^2\hat{\gamma}_{k,i}(1-\rho_k^2)/2) - r_i}{\sqrt{\frac{1}{m}(1-2^{-2\mathcal{C}(x^2\hat{\gamma}_{k,i}(1-\rho_k^2)/2)})}\log_2 e}$ and $x_{k,i} = \sqrt{\frac{2\hat{\gamma}_{k,i}}{\bar{\gamma}_k(1-\rho_k^2)}}$.

Notice that for the relaying system considered, the (source-to-destination) equivalent coding rate during each frame i is actually $r_i/2$. Therefore, the expected FBL throughput of relaying during frame i, i.e., the expected effectively transmitted information (number of correctly received bits at the destination) per channel use, is given by:

$$\mu_{\text{FBL},i} = \mathcal{C}_{\text{FBL}}(r_i) = r_i(1 - \bar{\varepsilon}_{\text{R},i}(r_i))/2. \quad (10)$$

The above $\mu_{\text{FBL},i}$ is the expected FBL throughput of relaying for an upcoming frame i based on a scheduled coding rate. By marginalizing over all possible channel states for both links, we finally end up with the average FBL throughput of relaying: $\mu_{\text{FBL}} = \mathop{\mathbb{E}}_{i=1,\ldots,+\infty}[\mathcal{C}_{\text{FBL},i}(r_i)] = \mathop{\mathbb{E}}_{\gamma_{1,i},\gamma_{2,i}}[\mathcal{C}_{\text{FBL},i}(r_i)]$. Note in particular that this throughput depends on the scheduled coding rate r_i which itself can be based on the information at hand of the source, i.e. the outdated SNRs $\hat{\gamma}_{1,i}$ and $\hat{\gamma}_{2,i}$.

B. Optimal Scheduling

Recall that we are interested in scenarios with reliability constraints, i.e., the (expected/average) error probability of each link should be lower than a threshold ε_{th} of practical interest, e.g., $\varepsilon_{\text{th}} \ll 0.5$. In the following, we study the optimal scheduling policy of determining the coding rate while guaranteeing the reliability constraint ε_{th}. If the source

[1]Note that only a single coding rate is scheduled for both links per frame.

$$\begin{aligned}\bar{\varepsilon}_{k,i} &= \int_0^{+\infty} \mathbb{P}\left[\gamma_{k,i}|\hat{\gamma}_{k,i}\right] \mathcal{P}(\gamma_{k,i},r_i,m) d\gamma_{k,i} = \int_0^{+\infty} \frac{I_0(\frac{2\rho_k\sqrt{\gamma_{k,i}\hat{\gamma}_{k,i}}}{\bar{\gamma}_k(1-\rho_k^2)})\exp(-\frac{\gamma_{k,i}+\rho_k^2\hat{\gamma}_{k,i}}{\bar{\gamma}_k(1-\rho_k^2)})}{\bar{\gamma}_k(1-\rho_k^2)} Q(\frac{\mathcal{C}(\gamma_{k,i})-r_i}{\sqrt{\frac{1}{m}\left(1-2^{-2\mathcal{C}(\gamma_{k,i})}\right)}\log_2 e}) d\gamma_{k,i} \\ &= \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} \int_{\alpha(\gamma_{k,i},r_i)}^{\infty} \frac{I_0(\frac{2\rho_k\sqrt{\gamma_{k,i}\hat{\gamma}_{k,i}}}{\bar{\gamma}_k(1-\rho_k^2)})}{\bar{\gamma}_k(1-\rho_k^2)} e^{-\frac{\gamma_{k,i}+\rho_k^2\hat{\gamma}_{k,i}}{\bar{\gamma}_k(1-\rho_k^2)}-\frac{t^2}{2}} dt d\gamma_{k,i} \overset{x=\sqrt{\frac{2\gamma_{k,i}}{\bar{\gamma}_k(1-\rho_k^2)}}}{=} \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} \int_{\alpha(x,r_i)}^{\infty} x I_0(\frac{x\rho_k\sqrt{2\hat{\gamma}_{k,i}}}{\sqrt{\bar{\gamma}_k(1-\rho_k^2)}}) e^{-\frac{x^2}{2}-\frac{\rho_k^2\hat{\gamma}_{k,i}}{\bar{\gamma}_k(1-\rho_k^2)}-\frac{t^2}{2}} dt dx \\ &= \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} \int_{\alpha(x,r_i)}^{\infty} x I_0\left(x\rho_k x_{k,i}\right) e^{-\frac{x^2+\rho_k^2 x_{k,i}+t^2}{2}} dt dx.\end{aligned} \tag{9}$$

schedules the coding rate directly based on the outdated CSI, it is likely that the (real) SNR is lower. According to the FBL model, this determined coding relate introduces a significant probability of error in each transmission, which perhaps violates the reliability constraint. To improve and guarantee the reliability, we introduce weights, i.e., SNR back-offs, to let the source choose a relatively lower coding rate obtained by scaling the outdated SNR. Denote these weights for frame i for the backhaul link by $\eta_{1,i}$ and for the relaying link by $\eta_{2,i}$, where $0 < \eta_{k,i}, k = 1, 2$. Recall that the performance of the two-hop relaying system is subject to the bottleneck link which can be either the backhaul or the relaying link. Thus, for a given selection of the weights η_i the coding rate r_i of frame i is determined based on the bottleneck link: $r_i = \mathcal{R}(\min\{\eta_{1,i}\hat{\gamma}_{1,i}, \eta_{2,i}\hat{\gamma}_{2,i}\}, \varepsilon_{\rm th}, m)$. It should be mentioned that setting relatively high weight values results in a higher coding rate. On the one hand, a high coding rate results in a high error probability. On the other hand, if the transmission is successful, a high coding rate indicates that more bits are transmitted (than setting a low coding rate). According to (10), both the error probability and the coding rate influence the throughput significantly. Thus, the proposed weights actually introduce a tradeoff in maximizing the throughput via scheduling the coding rate.

Our aim is to determine - per frame - the optimal scheduling weights (of the backhaul link and the relaying link) for coding rate scheduling which maximizes the average FBL throughput while guaranteeing the reliability of transmissions. Therefore, the optimization problem actually equals to maximize the expected FBL throughput per frame by solving the following optimization problem:

$$\begin{aligned} \max_{\eta_{1,i},\eta_{2,i}} \quad & \mu_{\rm FBL} \\ s.t.: \quad & \bar{\varepsilon}_{k,i} \leq \varepsilon_{\rm th}, k = 1, 2; i = 1, ..., +\infty. \end{aligned} \tag{11}$$

For this optimization problem, note that the space of feasible solutions for the scheduling weights is restricted in the following way: We are interested in reliable transmission, i.e. we restrict the transmission to the reliability constraint $\epsilon_{\rm th} \ll 0.5$. Then, according to (3) we have $\bar{\varepsilon}_{k,i} = \mathop{\mathbb{E}}_{\gamma_{k,i}|\hat{\gamma}_{k,i}}[\varepsilon_{k,i}] \leq 0.5 \Leftrightarrow \mathbb{P}\{\mathop{\mathbb{E}}_{\gamma_{k,i}|\hat{\gamma}_{k,i}}[\mathcal{C}(\gamma_{k,i})] \geq r_i\} \geq 0.5 \Leftrightarrow \mathbb{P}\{\gamma_{k,i} \geq \eta_{k,i}\hat{\gamma}_{k,i}\} \geq 0.5 \Leftrightarrow \eta_{k,i} \leq \rho_k^2$, which results in the space $\eta_{k,i} \in \left[0, \rho_k^2\right]$.

Under this constraint, the following proposition can be shown with respect to the scheduling of the weights for the considered relay system:

Proposition 1. *For a relay network operating on outdated CSI, if the coding rate for frame i is scheduled according to $r_i = \mathcal{R}(\min\{\eta_{1,i}\hat{\gamma}_{1,i}, \eta_{2,i}\hat{\gamma}_{2,i}\}, \varepsilon_{\rm th}, m)$, $\eta_{k,i} \in (0, \rho_k^2], k = 1, 2$, the expected FBL throughput of the upcoming frame i, $\mu_{{\rm FBL},i} = \mathcal{C}_{\rm FBL}(r_i)$, is concave in the coding rate r_i.*

Proof. See Appendix A. □

Recall that the coding rate is chosen by the source based on $\min\{\eta_{1,i}\hat{\gamma}_{1,i}, \eta_{2,i}\hat{\gamma}_{2,i}\}$. Due to (4), the coding rate is strictly increasing in $\min\{\eta_{1,i}\hat{\gamma}_{1,i}, \eta_{2,i}\hat{\gamma}_{2,i}\}$ and therefore increasing in $\eta_{1,i}$ or $\eta_{2,i}$. In combination with Proposition 1, we thus obtain an important corollary regarding the optimal scheduling of the system:

Corollary 1. *For a relay network operating on outdated CSI, if the coding rate for frame i is scheduled according to $r_i = \mathcal{R}(\min\{\eta_{1,i}\hat{\gamma}_{1,i}, \eta_{2,i}\hat{\gamma}_{2,i}\}, \varepsilon_{\rm th}, m)$, $\eta_{k,i} \in (0, \rho_k^2], k = 1, 2$, $\mathcal{C}_{{\rm FBL},i}$ the expected FBL throughput of frame i is quasi-concave in $\eta_{1,i}$ in the region $(0, \rho_1^2]$ and quasi-concave in $\eta_{2,i}$ in the region $(0, \rho_2^2]$.*

Proof. See Appendix B. □

According to Corollary 1, $C_{{\rm FBL},i}$ can be optimized by applying quasi-convex optimization techniques, e.g., backtracking line search, to obtain the optimal weights for determining the coding rate. Nevertheless, this can be computationally heavy, as this optimization step needs to be conducted prior to each data transmission. Note in this context that the smaller the reliability requirement is, the smaller is also the search space of the scheduling weights, making it more likely that for a given instance the optimal solution is on the boundary of the feasible set. Still, in order to reach the optimal system performance, some computations need to be executed prior to each frame.

C. Constant Weight Heuristic

To further reduce the computational complexity, in this section we consider a scheduling scheme where the scheduling weights are not adapted per frame. Once the scheduling weights are determined at system initialization (depending on the average SNR and the channel correlation coefficients) they remain constant during all frames. We are then interested in determining the constant heuristic with the best performance.

Denote these constant weights by η_1 and η_2 for the backhaul and relaying link. Then, the coding rate for frame i under the constant weight scheme is subject to the instantaneous SNR and the constant weights. As a result, obviously the coding rate is not constant over different frames. In particular, the coding rate r_i of frame i is obtained by: $r_i = \mathcal{R}(\min\{\eta_1\hat{\gamma}_{1,i}, \eta_2\hat{\gamma}_{2,i}\}, \varepsilon_{\rm th}, m)$. According to (4), the coding rate r_i is strictly increasing in $\min\{\eta_1\hat{\gamma}_{1,i}, \eta_2\hat{\gamma}_{2,i}\}$ and therefore monotonically increasing in η_1 and η_2. Thus, under this constant weight scheme, the average FBL throughput over Rayleigh fading channels is determined by:

$$\begin{aligned}
&\mu_{\rm FBL}(\eta_1,\eta_2) = \mathop{\mathbb{E}}_{r_i}\left[\mathcal{C}_{\rm FBL}(r_i)\right] \\
&= \int_0^\infty\int_0^\infty \mathcal{C}_{\rm FBL}\left(\mathcal{R}(\min\{\eta_1\hat{\gamma}_1,\eta_2\hat{\gamma}_2\},\varepsilon_{\rm th},m)\right)e^{-\frac{\hat{\gamma}_1}{\bar{\gamma}_1}-\frac{\hat{\gamma}_2}{\bar{\gamma}_2}}d\frac{\hat{\gamma}_1}{\bar{\gamma}}d\frac{\hat{\gamma}_2}{\bar{\gamma}} \\
&= \frac{1}{\bar{\gamma}_1\bar{\gamma}_2}\int_0^\infty\int_{\frac{\eta_1\hat{\gamma}_1}{\eta_2\hat{\gamma}_2}}^\infty \mathcal{C}_{\rm FBL}\left(\mathcal{R}(\eta_1\hat{\gamma}_1,\varepsilon_{\rm th},m)\right)e^{-\frac{\hat{\gamma}_1}{\bar{\gamma}_1}-\frac{\hat{\gamma}_2}{\bar{\gamma}_2}}d\hat{\gamma}_2 d\hat{\gamma}_1 \\
&\quad + \frac{1}{\bar{\gamma}_1\bar{\gamma}_2}\int_0^\infty\int_{\frac{\eta_2\hat{\gamma}_2}{\eta_1\hat{\gamma}_1}}^\infty \mathcal{C}_{\rm FBL}(\mathcal{R}(\eta_2\hat{\gamma}_2,\varepsilon_{\rm th},m))e^{-\frac{\hat{\gamma}_1}{\bar{\gamma}_1}-\frac{\hat{\gamma}_2}{\bar{\gamma}_2}}d\hat{\gamma}_1 d\hat{\gamma}_2.
\end{aligned} \tag{12}$$

Under the best constant heuristic, the aim is to maximize the average FBL throughput while the constraint is to guarantee the average error probability over time [2]. Then, the resulting optimization problem is given by:

$$\begin{aligned}
&\max_{\eta_1,\eta_2} \quad \mu_{\rm FBL}(\eta_1,\eta_2). \\
&s.t: \quad \mathop{\mathbb{E}}_{\gamma_{k,i}}\left[\bar{\varepsilon}_{k,i}\right] \le \varepsilon_{\rm th}, k=1,2.
\end{aligned} \tag{13}$$

As we assume $\varepsilon_{\rm th} \ll 0.5$, we have $\mathop{\mathbb{E}}_{\gamma_{k,i}}[\bar{\varepsilon}_{k,i}] = \mathop{\mathbb{E}}_{\hat{\gamma}_{k,i}}[\mathop{\mathbb{E}}_{\gamma_{k,i}|\hat{\gamma}_{k,i}}[\varepsilon_{k,i}]] \le 0.5 \Leftrightarrow \mathop{\mathbb{E}}_{\hat{\gamma}_{k,i}}[\mathop{\mathbb{E}}_{\gamma_{k,i}|\hat{\gamma}_{k,i}}[Q(\frac{\mathcal{C}(\gamma_{k,i})-r_i}{\sqrt{V/m}})]] \le 0.5 \Leftrightarrow \mathop{\mathbb{E}}_{\hat{\gamma}_{k,i}}[\mathbb{P}\{\mathop{\mathbb{E}}_{\gamma_{k,i}|\hat{\gamma}_{k,i}}[\mathcal{C}(\gamma_{k,i})] \ge r_i\}] \ge 0.5 \Leftrightarrow \mathop{\mathbb{E}}_{\hat{\gamma}_{k,i}}[\mathbb{P}\{\gamma_{k,i} \ge \eta_k\hat{\gamma}_{k,i}\}] \ge 0.5 \Leftrightarrow \eta_k \le \rho_k^2$. Therefore, the feasible set of $\eta_{k,i}$ is $(0,\rho_k^2]$ under the case $\varepsilon_{\rm th} = 0.5$ and covers a subset of $(0,\rho_k^2]$ when $\varepsilon_{\rm th} \ll 0.5$.

Denote η_1^* and η_2^* as the solution to the above optimization problem, i.e. they are the optimal, constant scheduling weights. We then have the following proposition:

Proposition 2. *Considering a relay network operating on outdated CSI with constant scheduling weights, if the coding rate of each frame i is scheduled according to $r_i = \mathcal{R}(\min\{\eta_1\hat{\gamma}_{1,i}, \eta_2\hat{\gamma}_{2,i}\}, \varepsilon_{\rm th}, m)$, then the average FBL throughput $C_{\rm FBL}$ is quasi-concave in η_1 in the region $(0,\rho_1^2]$ and quasi-concave in η_2 in the region $(0,\rho_2^2]$.*

Proof. See Appendix C. □

According to Proposition 2, (13) can be efficiently solved by applying quasi-convex optimization techniques. For a relay system with a certain set of average SNRs and correlation coefficients as well as a given reliability constraint, we obtain a unique pair of fixed scheduling weights. Note that these fixed weights are then strictly applied per frame, leading to a varying coding rate that maximizes the long-term average FBL throughput (under the assumption of using fixed weights). This reduces drastically the computational complexity, but leads to an inferior system performance in comparison to the optimal scheduling scheme with adaptive scheduling weights, i.e. the optimal solution presented in Section III-B.

[2]From a statistical point of view, guaranteeing the expected error probability per frame leads to the same results as guaranteeing the average error probability over time

IV. Numerical Evaluation and Discussion

In this section, we present numerical results regrading the FBL throughput maximization in the considered relay network operating with outdated CSIs. We consider in particular two issues: Initially, we study several aspects of the quasi-convexity of the FBL throughput with respect to the scheduling weights. In particular, we are interested in the sharpness of the optimum. This investigation is important for practical system design, as it clarifies the potential cost of non-optimal weight selection. After clarifying these issues, we move to a more general performance investigation. Here, we are especially interested in the performance comparison between the optimal scheduling scheme (with changing scheduling weights per frame) and the low-complexity best constant heuristic.

From a methodological point of view, all following numerical results are based on simulations. We consider a basic scenario for these simulations with the following parameterization: We assume an urban outdoor scenario where the distances of the backhaul and relaying link are both set to 100 $\rm m$. For channel propagation, we utilize the well-known COST [31] model (which is a commonly-used model for urban scenarios) for calculating the path loss. The center frequency is set to 2 GHz while the transmit power $p_{\rm tx}$ is selected to 35 $\rm dBm$ (we vary the transmit power in Figures 7 and 8) considering a noise power of -90 $\rm dBm$, respectively. Lastly, the blocklength at each hop of relaying is set to $m = 300$ symbols[3]. Recall that the channel correlation coefficients ρ_1^2 and ρ_1^2 of the backhaul and relaying links are subject to the settings of n, $n+m$ and the Doppler frequency. In particular, $\rho_1^2 \ge \rho_2^2$ as $n \le n+m$, i.e., the CSI of the relaying phase is more delayed. In the simulation, we don't set a fixed value for either the length of initialization phase n or the Doppler frequency. Instead, we consider different setups of ρ_1^2 and ρ_1^2, which corresponds to different settings of n and the Doppler frequency as m is fixed, while $\rho_1^2 \ge \rho_2^2$ holds for all setups.

A. Quasi-Convexity of the FBL Throughput

In this subsection, we consider numerical results regarding the quasi-convexity of the average FBL throughput. We first study the relationship between the expected FBL throughput of an upcoming frame and the choice of scheduling weights (based on the corresponding outdated CSI) in case of the optimal scheduling scheme that adapts the weights per frame. In order to do so, we fix the outdated CSI and generate realizations of the corresponding instantaneous channel states. Then, we study the expected FBL throughput (the expectation/average over all realizations) by varying the scheduling weights. The results are shown in Figure 3. First of all, the figure illustrates that the FBL throughput per frame is quasi-concave in the scheduling weights $\eta_{1,i}$ and $\eta_{2,i}$. Hence, by choosing appropriate values for $\eta_{1,i}$ and $\eta_{2,i}$ the FBL throughput can be optimized. Secondly, the figure also shows that the expected FBL throughput of the upcoming frame is actually subject to both scheduling weights $\eta_{1,i}$ and $\eta_{2,i}$ in

[3]From [8, Figure 2] it is known that the relative difference of the approximate and the exact achievable rates is less than 2% for cases with $m \ge 100$.

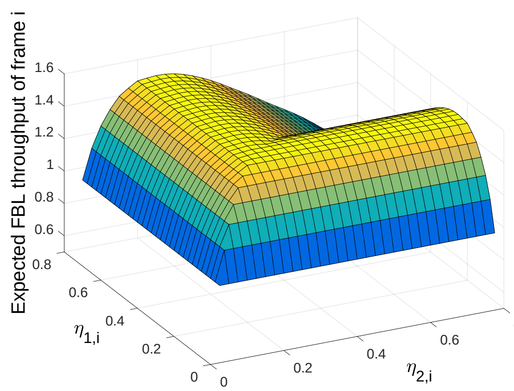

Fig. 3. Expected FBL throughput [bit/ch.use] of an upcoming frame i vs. the choice of scheduling weights. In the figure, we set $\rho_1^2 = 0.7$ and $\rho_2^2 = 0.5$.

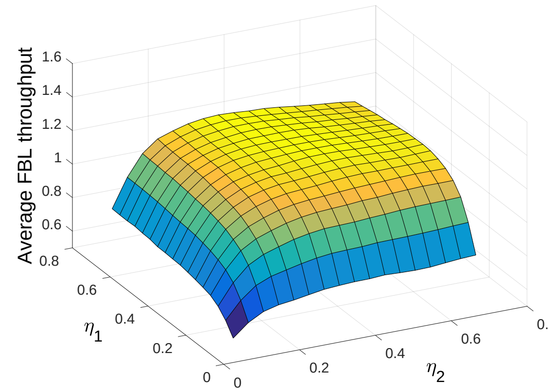

Fig. 5. Average FBL throughput [bit/ch.use] vs. choice of constant scheduling weights for a relay system with parameters $\rho_1^2 = 0.7$ and $\rho_2^2 = 0.5$.

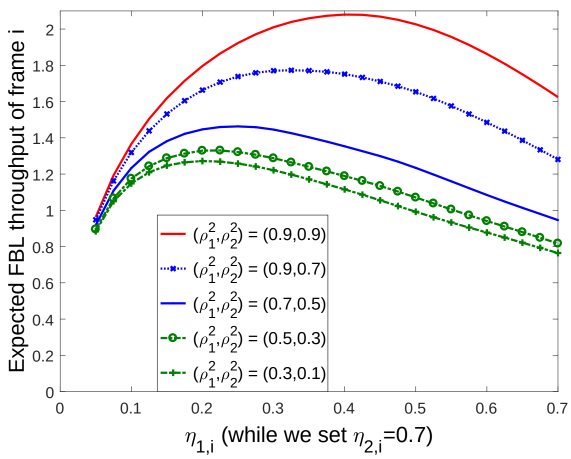

Fig. 4. Expected FBL throughput [bit/ch.use] of an upcoming frame i with different channel correlation coefficients. In the figure, we vary the scheduling weight $\eta_{1,i}$ while setting $\eta_{2,i} = 0.7$.

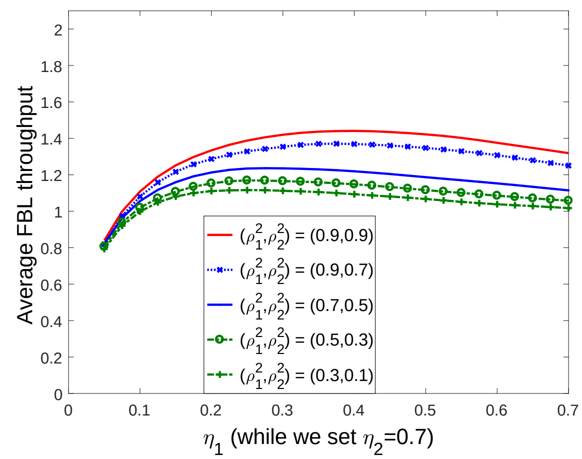

Fig. 6. Average FBL throughput [bit/ch.use] vs. choice of constant scheduling weights for relay systems with different channel correlation coefficients. In the figure, we vary the fixed scheduling weight η_1 while setting $\eta_2 = 0.7$.

general. However, if $\eta_{1,i}$ is chosen optimally, then the choice of $\eta_{2,i}$ can be arbitrarily large (but not arbitrarily small). This stems essentially from the fact how the scheduling weights influence the bottleneck link. The case "FBL throughput being only influenced by $\eta_{1,i}$" corresponds to the situation where the bottleneck link (for determining the coding rate) is the backhaul link. At the same time, as long as $\eta_{2,i}$ (the scheduling weight of the relaying link) is not set to a very small value, the impact on the SNR of the backhaul link is considerably small and therefore does not influence the coding rate. In other words, there is no impact of a link's scheduling weight on the FBL throughput of the upcoming frame if this link is not the bottleneck link. Obviously, reducing the scheduling weight of a link likely makes this link become the bottleneck link eventually. As a consequence of this dependence between $\eta_{1,i}$ and $\eta_{2,i}$, we observe that there are multiple solutions maximizing the FBL throughput surface for the considered channel setting.

We next study the quasi-convexity of the optimal scheduling for scenarios with different channel correlation setups. The results are shown in Figure 4 where we fix $\eta_{2,i}$ to 0.7, i.e., make the backhaul link the bottleneck and vary $\eta_{1,i}$. The figure reveals that a stronger channel correlation results in a higher optimal FBL throughput. More importantly, this higher maximum is achieved by a bigger scheduling weight. In other words, a strong channel correlation allows us to set the scheduling weight more aggressively, leading to a higher coding rate and a higher FBL throughput.

We now turn to the constant heuristic, where the scheduling weight is determined once for the given system and then left constant for each frame. In Figure 5, we show the relationship between the average FBL throughput $\mathcal{C}_{\mathrm{BL}}$ and the constant scheduling weights η_1 and η_2 while generating many different outdated channel instances and the corresponding instantaneous channel state realizations. Firstly, the figure confirms again our analytical insight (Proposition 2), i.e. C_{FBL} is quasi-concave in η_1 or η_2. In addition, we observe that a near-optimal FBL throughput is achieved for a large set of different scheduling weights, e.g., a small error of the optimal solution does not change the average FBL throughput too much. Hence, the FBL throughput in the case of the constant scheduling weights is somewhat robust to an erroneous choice of the weights. Similar to the optimal scheduling, we further study the average FBL throughput of the best constant heuristic with different channel correlation coefficients. The results are provided in Figure 6. It is shown that under the best constant heuristic scheme a strong channel correlation also introduces a higher FBL throughput attained for larger scheduling weights.

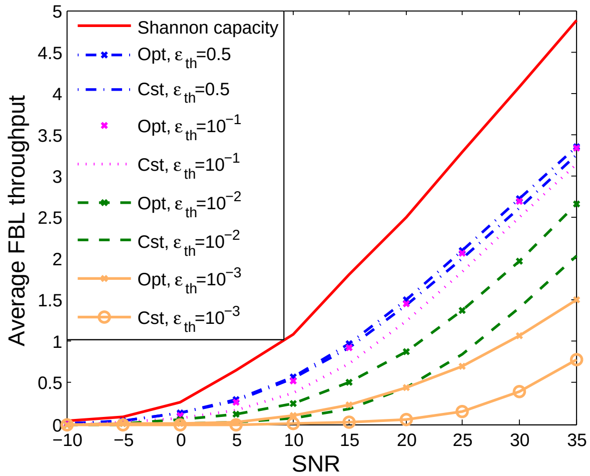

Fig. 7. Comparison of the average FBL throughput [bit/ch.use] for the two different schemes (optimal and constant scheduling) versus the average channel SNR for different reliability constraints $\varepsilon_{\mathrm{th}}$. For the channel correlations, we considered $\rho_1^2 = 0.7$ and $\rho_2^2 = 0.5$.

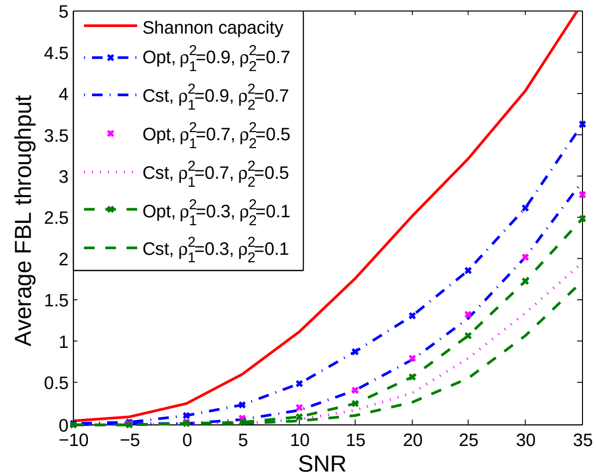

Fig. 8. Comparison of the average FBL throughput [bit/ch.use] for the two different schemes (optimal and constant scheduling) versus the average channel SNR for different correlation coefficients of the links. For the reliability constraint we set $\varepsilon_{\mathrm{th}} = 10^{-2}$.

Nevertheless, note that the throughput difference is smaller when comparing the throughput for small and large channel correlations in case of the constant scheduling weights in comparison to the optimal, adaptive choice of the scheduling weights per frame (Figure 4).

We conclude the discussion regarding the quasi-convexity by summarizing the following guidelines for choosing the scheduling weights: Firstly, in general the optimal weights are lower than 0.5 even for channels with high correlation coefficients. Secondly, in comparison to a weak channel correlation, a strong one allows us to set relatively higher scheduling weights. Thirdly, it is important to have an accurate characterization of the channel correlation, otherwise the FBL throughput can be significantly reduced. In particular, the optimal scheduling scheme is more sensitive to an inaccurate knowledge of the channel correlations than the constant weight heuristic. Furthermore, a low error probability constraint leads to a small feasible set for choosing the scheduling weights. Finally, it appears that for constant weight scheduling, the choice of the weights is less sensitive to wrong choices, especially if these choices end up being too large. In the case of the optimal scheduling, this only applies to cases where one of the link weights is set optimally.

B. Optimal vs. Constant Scheduling

In this subsection, we focus on investigating the performance gap between the two schemes presented in Section III-B and III-C for a set of variable parameters with respect to the SNR, reliability constraint and channel correlation coefficient. To start with, we show the FBL throughput of the two schemes versus the average channel SNR while considering different settings of the reliability threshold $\varepsilon_{\mathrm{th}}$. The results are shown in Figure 7, where the average FBL throughputs are based on the optimal/sub-optimal choice of coding rate under either the optimal scheduling or the best constant heuristic. Firstly, we observe that the lower the reliability threshold is, the lower the optimal average FBL throughput is. Secondly, a higher SNR also increases the gap between the optimal scheduling and the constant heuristic. Lastly, a lower reliability constraint $\varepsilon_{\mathrm{th}}$ leads to a significantly bigger gap between the two schemes. For instance, the gap is quite big under the constraint $\varepsilon_{\mathrm{th}} = 10^{-3}$ while it is small when $\varepsilon_{\mathrm{th}} = 0.5$. This suggests that it only pays off to spend the additional computational complexity for the optimal scheduling scheme in case of a high reliability constraint (i.e. a rather low requirement on the error probability). In case of a rather low reliability constraint, there is no big difference between the two scheduling schemes. This is essentially due to the fact that in case of a high reliability constraint the constant heuristic needs to select a proportionally lower value for the scheduling weight to fulfill the reliability constraint even in cases where the instantaneous channel state drops significantly below the outdated CSI. In case of the optimal scheduling, this can be compensated for by frame-specific scheduling of the weights.

Finally, we show in Figure 8 the average FBL throughput of the two different schemes while considering different channel correlation coefficients. In particular, the reliability constraint in the figure is set to be fixed as $\varepsilon_{\mathrm{th}} = 10^{-2}$. We observe that there is a big loss in comparison to the Shannon capacity, even for the FBL throughput with a strong channel correlation. In addition, a stronger channel correlation introduces a higher FBL throughputs for both the optimal scheduling and the constant heuristic. Furthermore, in the high SNR region the performance gap between the two schemes is less influenced by the channel correlation, e.g., at point SNR=25 dB the gap between the two schemes of the case $(\rho_1^2 = 0.9, \rho_2^2 = 0.7)$ is quite similar to the gaps under the other two cases. This is due to the fact that a strong channel correlation makes the outdated CSI more accurate, which reduces the importance of choosing the best scheduling weight. As a result, the performance gap between the optimal and the heuristic scheduling schemes is relatively constant. On the other hand, in the low SNR region the gap between the two schemes is slightly bigger for the case with a strong channel correlation.

Combining the insights from Figure 7 and Figure 8, we can conclude that the performance gap between the proposed optimal scheme and the heuristic scheduling scheme mainly depends on the error probability threshold and the average channel SNR, while it is only marginally influenced by the channel correlation coefficients. This indicates that it is perhaps only worth to spend the computational complexity of the optimal scheme in case of high reliability constraints and a rather high average SNR.

V. Conclusion

In this work, we study the finite blocklength performance of relaying with outdated CSI. Both an optimal and an low-complexity sub-optimal scheduling scheme are proposed to maximize the FBL throughput while satisfying a reliability constraint regarding the data transmission. We show that in both cases an optimal performance can be obtained by exploiting the quasi-concavity of the FBL throughput with respect to scheduling weights. By numerical analysis, we conclude a set of guidelines for the design of efficient relaying systems in the FBL regime. Firstly, it is important to have accurate channel correlation information, otherwise the inaccurate channel correlation coefficients can reduce the throughput. In particular, the optimal scheme is more sensitive regarding the accuracy of the channel correlation. Secondly, the optimal scheme is more sensitive to erroneous selection of the scheduling weights in comparison to the constant scheme. Thus, in practice a precise computation of the scheduling weights in case of the optimal scheme needs to be performed which nevertheless only pays off in certain scenarios. For the constant scheme, a less accurate computation of the optimal weights leads already to a satisfactory performance in particular if the scheduling weights are chosen rather too large versus too small. Thirdly, the performance gap between the proposed two schemes depends mainly on the reliability constraint regarding the data transmissions. The stricter this constraint is, the more does the optimal scheme outperform the constant scheme. Moreover, the performance gap between the two schemes is less influenced by the channel correlation coefficients.

Although in this work we assumed a block-fading Rayleigh channel, our results can be easily extended to other fading models, e.g., Rice fading. Then, the general FBL throughput model and the problem structure in this work still hold for the network with a new channel fading model. In particular, the optimal scheduling scheme, which adjusts the scheduling weights per frame (or per channel realization), can be directly applied under the new fading model. On the other hand, the constant weight heuristic can be applied after recalculating the objective provided in (12) by averaging the instantaneous throughput over the channel gain distribution of the new model. However, note that the Rayleigh fading is known as one of the most pessimistic fading models. As we have shown the critical FBL communication is possible for relaying under Rayleigh fading channels, the considered relaying network will achieve a better FBL performance under a assumption with better channels for the two hops of relaying (e.g., under a Rice channel fading model, the channel at each hop is assumed to have a line-of-sight component).

Appendix A
Proof of Proposition 1

Based on (8) and (10), we immediately have $\mu_{\mathrm{FBL},i} = \mathcal{C}_{\mathrm{FBL}}(r_i) = (1-\bar{\varepsilon}_{1,i}(r_i))(1-\bar{\varepsilon}_{2,i}(r_i))r_i/2$. Therefore, the first and second derivatives of $\mathcal{C}_{\mathrm{FBL}}$ with respect to r_i are given by:

$$\frac{\partial \mathcal{C}_{\mathrm{FBL}}}{\partial r_i} = \frac{(1-\bar{\varepsilon}_{1,i}(r_i))(1-\bar{\varepsilon}_{2,i}(r_i))}{2} - \frac{\partial \bar{\varepsilon}_{1,i}}{\partial r_i}\frac{(1-\bar{\varepsilon}_{2,i}(r_i))r_i}{2} - \frac{\partial \bar{\varepsilon}_{2,i}}{\partial r_i}\frac{(1-\bar{\varepsilon}_{1,i}(r_i))r_i}{2}, \tag{14}$$

$$\begin{aligned}\frac{\partial^2 \mathcal{C}_{\mathrm{FBL}}}{\partial^2 r_i} =& -\frac{\partial \bar{\varepsilon}_{1,i}}{\partial r_i}(1-\bar{\varepsilon}_{2,i}(r_i)) - \frac{\partial \bar{\varepsilon}_{2,i}}{\partial r_i}(1-\bar{\varepsilon}_{1,i}(r_i)) \\ & - \frac{\partial^2 \bar{\varepsilon}_{1,i}}{\partial^2 r_i}\frac{(1-\bar{\varepsilon}_{2,i}(r_i))r_i}{2} - \frac{\partial^2 \bar{\varepsilon}_{2,i}}{\partial^2 r_i}\frac{(1-\bar{\varepsilon}_{1,i}(r_i))r_i}{2} \\ & + \frac{\partial \bar{\varepsilon}_{1,i}}{\partial r_i}\frac{\partial \bar{\varepsilon}_{2,i}}{\partial r_i} r_i.\end{aligned} \tag{15}$$

In the following, we prove Proposition 1 by showing $\frac{\partial^2 \mathcal{C}_{\mathrm{FBL}}}{\partial^2 r_i} < 0$.

Recall that $\varepsilon_{k,i} = \mathcal{P}(\gamma_{k,i}, r_i, m), k = 1,2$. According to (4), we have:

$$\frac{\partial \varepsilon_{k,i}}{\partial r_i} = \frac{m^{\frac{1}{2}} \exp\left(-\frac{m(\mathcal{C}(\gamma_{k,i})-r_i)^2}{2(1-2^{-2\mathcal{C}(\gamma_{k,i})})(\log_2 e)^2}\right)}{\sqrt{2\pi}\left(1-2^{-2\mathcal{C}(\gamma_{k,i})}\right)^{\frac{1}{2}}\log_2 e} > 0, \tag{16}$$

$$\frac{\partial^2 \varepsilon_{k,i}}{\partial^2 r_i} = \frac{m^{\frac{3}{2}}\left(\mathcal{C}(\gamma_{k,i})-r_i\right) \exp\left(-\frac{m(\mathcal{C}(\gamma_{k,i})-r_i)^2}{2(1-2^{-2\mathcal{C}(\gamma_{k,i})})(\log_2 e)^2}\right)}{\sqrt{2\pi}(1-2^{-2\mathcal{C}(\gamma_{k,i})})^{\frac{3}{2}}(\log_2 e)^3}. \tag{17}$$

Based on (9), we have:

$$\frac{\partial \bar{\varepsilon}_{k,i}}{\partial r_i} = \int_0^{+\infty} \mathbb{P}\left[\gamma_{k,i}|\hat{\gamma}_{k,i}\right]\frac{\partial \varepsilon_{k,i}}{\partial r_i} d\gamma_{k,i} > 0, \tag{18}$$

$$\frac{\partial^2 \bar{\varepsilon}_{k,i}}{\partial^2 r_i} = \int_0^{+\infty} \mathbb{P}\left[\gamma_{k,i}|\hat{\gamma}_{k,i}\right]\frac{\partial^2 \varepsilon_{k,i}}{\partial^2 r_i} d\gamma_{k,i}. \tag{19}$$

As $\eta_k \leq \rho_k^2, k = 1,2$, the following inequality holds: $r_i < \mathcal{C}\left(\min_{k=1,2}\{\eta_k\hat{\gamma}_{k,i}\}\right) \leq \mathcal{C}\left(\min_{k=1,2}\{\rho_k^2\hat{\gamma}_{k,i}\}\right)$. Hence, $\mathcal{C}(\gamma_{k,i}) - r_i > 0$ and therefore $\frac{\partial^2 \varepsilon_{k,i}}{\partial^2 r_i} > 0$ during the intervals $\gamma_{k,i} \in [\eta_k\hat{\gamma}_{k,i}, \rho_k^2\hat{\gamma}_{k,i})$ and $\gamma_{k,i} \in [\rho_k^2\hat{\gamma}_{k,i} + \infty)$. As $\mathbb{P}\left[\gamma_{k,i}|\hat{\gamma}_{k,i}\right] > 0$, hence we have: $\int_{\eta_k\hat{\gamma}_{k,i}}^{\rho_k^2\hat{\gamma}_{k,i}} \mathbb{P}\left[\gamma_{k,i}|\hat{\gamma}_{k,i}\right]\frac{\partial^2 \varepsilon_{k,i}}{\partial^2 r_i} d\gamma_{k,i} > 0$ and $\int_{\rho_k^2\hat{\gamma}_{k,i}}^{+\infty} \mathbb{P}\left[\gamma_{k,i}|\hat{\gamma}_{k,i}\right]\frac{\partial^2 \varepsilon_{k,i}}{\partial^2 r_i} d\gamma_{k,i} > 0$.

Considering equation (3), we have:

$$\begin{aligned}&\int_{\rho_k^2\hat{\gamma}_{k,i}}^{+\infty} \mathbb{P}\left[\gamma_{k,i}|\hat{\gamma}_{k,i}\right]\frac{\partial^2 \varepsilon_{k,i}}{\partial^2 r_i} d\gamma_{k,i} \\ =& \int_{\rho_k^2\hat{\gamma}_{k,i}}^{+\infty} \mathbb{P}\left[\gamma_{k,i}|\hat{\gamma}_{k,i}\right]\frac{m^{\frac{3}{2}}\left(\mathcal{C}(\gamma_{k,i})-r_i\right) e^{-\frac{m(\mathcal{C}(\gamma_{k,i})-r_i)^2}{2(1-2^{-2\mathcal{C}(\gamma_{k,i})})(\log_2 e)^2}}}{\sqrt{2\pi}(1-2^{-2\mathcal{C}(\gamma_{k,i})})^{\frac{3}{2}}(\log_2 e)^3} d\gamma_{k,i} \\ >& \int_0^{\rho_k^2\hat{\gamma}_{k,i}} \mathbb{P}\left[\gamma_{k,i}|\hat{\gamma}_{k,i}\right]\frac{m^{\frac{3}{2}}\left|\mathcal{C}(\gamma_{k,i})-r_i\right| e^{-\frac{m(\mathcal{C}(\gamma_{k,i})-r_i)^2}{2(1-2^{-2\mathcal{C}(\gamma_{k,i})})(\log_2 e)^2}}}{\sqrt{2\pi}(1-2^{-2\mathcal{C}(\gamma_{k,i})})^{\frac{3}{2}}(\log_2 e)^3} d\gamma_{k,i} \\ >& \int_0^{\eta_k\hat{\gamma}_{k,i}} \mathbb{P}\left[\gamma_{k,i}|\hat{\gamma}_{k,i}\right]\frac{m^{\frac{3}{2}}\left|\mathcal{C}(\gamma_{k,i})-r_i\right| e^{-\frac{m(\mathcal{C}(\gamma_{k,i})-r_i)^2}{2(1-2^{-2\mathcal{C}(\gamma_{k,i})})(\log_2 e)^2}}}{\sqrt{2\pi}(1-2^{-2\mathcal{C}(\gamma_{k,i})})^{\frac{3}{2}}(\log_2 e)^3} d\gamma_{k,i} \\ =& \left|\int_0^{\eta_k\hat{\gamma}_{k,i}} \mathbb{P}\left[\gamma_{k,i}|\hat{\gamma}_{k,i}\right]\frac{\partial^2 \varepsilon_{k,i}}{\partial^2 r_i} d\gamma_{k,i}\right|.\end{aligned} \tag{20}$$

So far, it has been shown that

$$\begin{aligned}\frac{\partial^2\bar{\varepsilon}_{k,i}}{\partial^2 r_i} &= \int_0^{+\infty}\mathbb{P}\left[\gamma_{k,i}|\hat{\gamma}_{k,i}\right]\frac{\partial^2\varepsilon_{k,i}}{\partial^2 r_i}d\gamma_{k,i}\\ &\geq \int_{\eta_k\hat{\gamma}_{k,i}}^{\rho_k^2\hat{\gamma}_{k,i}}\mathbb{P}\left[\gamma_{k,i}|\hat{\gamma}_{k,i}\right]\frac{\partial^2\varepsilon_{k,i}}{\partial^2 r_i}d\gamma_{k,i}\\ &\quad+\int_{\rho_k^2\hat{\gamma}_{k,i}}^{+\infty}\mathbb{P}\left[\gamma_{k,i}|\hat{\gamma}_{k,i}\right]\frac{\partial^2\varepsilon_{k,i}}{\partial^2 r_i}d\gamma_{k,i}\\ &\quad-\left|\int_0^{\eta_k\hat{\gamma}_{k,i}}\mathbb{P}\left[\gamma_{k,i}|\hat{\gamma}_{k,i}\right]\frac{\partial^2\varepsilon_{k,i}}{\partial^2 r_i}d\gamma_{k,i}\right|\\ &>0\ .\end{aligned} \tag{21}$$

According to (5), the error probability of a link is higher than 0.5 only if the coding rate is higher than the Shannon capacity. Recall that the coding rate chosen by the source satisfies $r_i < \mathcal{C}\left(\min\limits_{k=1,2}\{\eta_k\hat{\gamma}_{k,i}\}\right) \leq \mathcal{C}\left(\min\limits_{k=1,2}\{\rho_k^2\hat{\gamma}_{k,i}\}\right)$. This makes the expected error probability of each single link (during frame i) be lower than 0.5, i.e., $\bar{\varepsilon}_{k,i} < 0.5, k=1,2$. Based on (15), we have:

$$\begin{aligned}\frac{\partial^2\mathcal{C}_{\rm FBL}}{\partial^2 r_i} &< -\frac{\partial^2\bar{\varepsilon}_{1,i}}{\partial^2 r_i}\frac{r_i}{4}-\frac{\partial^2\bar{\varepsilon}_{2,i}}{\partial^2 r_i}\frac{r_i}{4}+\frac{\partial\bar{\varepsilon}_{1,i}}{\partial r_i}\frac{\partial\bar{\varepsilon}_{2,i}}{\partial r_i}r_i\\ &<\left(2\frac{\partial\bar{\varepsilon}_{1,i}}{\partial r_i}\frac{\partial\bar{\varepsilon}_{2,i}}{\partial r_i}-\frac{\partial^2\bar{\varepsilon}_{1,i}}{\partial^2 r_i}-\frac{\partial^2\bar{\varepsilon}_{2,i}}{\partial^2 r_i}\right)\frac{r_i}{2}\\ &\leq\left(2\frac{\partial\bar{\varepsilon}_{1,i}}{\partial r_i}\frac{\partial\bar{\varepsilon}_{2,i}}{\partial r_i}-2\sqrt{\frac{\partial^2\bar{\varepsilon}_{1,i}}{\partial^2 r_i}\frac{\partial^2\bar{\varepsilon}_{2,i}}{\partial^2 r_i}}\right)\frac{r_i}{2}\ .\end{aligned} \tag{22}$$

Hence, $\frac{\partial^2\mathcal{C}_{\rm FBL}}{\partial^2 r_i} < 0$ if $\frac{\partial^2\bar{\varepsilon}_{k,i}}{\partial^2 r_i}-\left(\frac{\partial\bar{\varepsilon}_{k,i}}{\partial r_i}\right)^2 > 0$.

According to the Cauchy–Schwarz inequality, we have $\left(\frac{\partial\bar{\varepsilon}_{k,i}}{\partial r_i}\right)^2 = \{\int_0^{+\infty}\mathbb{P}\left[\gamma_{k,i}|\hat{\gamma}_{k,i}\right]\frac{\partial\varepsilon_{k,i}}{\partial r_i}d\gamma_{k,i}\}^2 \leq \int_0^{+\infty}\mathbb{P}\left[\gamma_{k,i}|\hat{\gamma}_{k,i}\right]^2(\frac{\partial\varepsilon_{k,i}}{\partial r_i})^2 d\gamma_{k,i} < \int_0^{+\infty}\mathbb{P}\left[\gamma_{k,i}|\hat{\gamma}_{k,i}\right](\frac{\partial\varepsilon_{k,i}}{\partial r_i})^2 d\gamma_{k,i}$.

Hence, we have:

$\frac{\partial^2\bar{\varepsilon}_{k,i}}{\partial^2 r_i}-\left(\frac{\partial\bar{\varepsilon}_{k,i}}{\partial r_i}\right)^2$
$> \int_0^{+\infty}\mathbb{P}\left[\gamma_{k,i}|\hat{\gamma}_{k,i}\right]\left[\frac{\partial^2\varepsilon_{k,i}}{\partial^2 r_i}-(\frac{\partial\varepsilon_{k,i}}{\partial r_i})^2\right]d\gamma_{k,i}$
$= \int_0^{+\infty}\frac{m\cdot e^A}{\sqrt{2\pi}(\log_2 e)^2\left(1-2^{-2\mathcal{C}(\gamma_{k,i})}\right)}\cdot\mathbb{P}\left[\gamma_{k,i}|\hat{\gamma}_{k,i}\right]Bd\gamma_{k,i}$,

where $A = (-\frac{m(\mathcal{C}(\gamma_{k,i})-r_i)^2}{2(1-2^{-2\mathcal{C}(\gamma_{k,i})})(\log_2 e)^2})$ and $B = \frac{m^{\frac{1}{2}}(\mathcal{C}(\gamma_{k,i})-r_i)}{\left(1-2^{-2\mathcal{C}(\gamma_{k,i})}\right)^{\frac{1}{2}}\log_2 e}-\frac{e^A}{\sqrt{2\pi}} < \frac{m^{\frac{1}{2}}(\mathcal{C}(\gamma_{k,i})-r_i)-\frac{\ln 2}{2}}{\left(1-2^{-2\mathcal{C}(\gamma_{k,i})}\right)^{\frac{1}{2}}\log_2 e}$. There exists a positive constant t, which makes $B \leq \frac{t\cdot m^{\frac{1}{2}}(\mathcal{C}(\gamma_{k,i})-r_i)}{\left(1-2^{-2\mathcal{C}(\gamma_{k,i})}\right)^{\frac{1}{2}}\log_2 e}$. Same to the discussion in (20) and (21), it holds that: $\int_0^{+\infty}\frac{m\cdot e^A\cdot\mathbb{P}[\gamma_{k,i}|\hat{\gamma}_{k,i}]}{\sqrt{2\pi}(\log_2 e)^2\left(1-2^{-2\mathcal{C}(\gamma_{k,i})}\right)}\frac{t\cdot m^{\frac{1}{2}}(\mathcal{C}(\gamma_{k,i})-r_i)}{\left(1-2^{-2\mathcal{C}(\gamma_{k,i})}\right)^{\frac{1}{2}}\log_2 e}d\gamma_{k,i} > 0$. Hence, $\frac{\partial^2\mathcal{C}_{\rm FBL}}{\partial^2 r_i} < 0$. As $\mu_{{\rm FBL},i} = \mathcal{C}_{\rm FBL}(r_i)$, $\mu_{{\rm FBL},i}$ is concave in r_i.

Appendix B
Proof of Corollary 1

The coding rate of frame i is scheduled based on the outdated CSI of the bottleneck link, i.e., $\min\{\eta_{1,i}\hat{\gamma}_{1,i},\eta_{2,i}\hat{\gamma}_{2,i}\}$. According to (4), we know that r_i is strictly increasing in $\min\{\eta_{1,i}\hat{\gamma}_{1,i},\eta_{2,i}\hat{\gamma}_{2,i}\}$. Hence, when the source schedules the coding rate r_i, high values of $\eta_{1,i}$ and $\eta_{2,i}$ lead to a big r_i. In other words, r_i is monotonically increasing in $\eta_{k,i}, k=1,2$.

$\Rightarrow \forall\ x_k < y_k$, $x_k, y_k \in (0,\rho_k]$ and $\lambda_k \in [0,1]$, we have $r_i|_{\eta_{k,i}=x_k} < r_i|_{\eta_{k,i}=\lambda_k x_k+(1-\lambda_k)y_k} < r_i|_{\eta_{k,i}=y_k}$, where $k=1,2$.

Based on Proposition 1, $\mathcal{C}_{\rm FBL}$ is concave in r_i,

$\Rightarrow \min\left\{\mathcal{C}_{{\rm BL},i}\left(r_i|_{\eta_{k,i}=x_k}\right),\mathcal{C}_{{\rm BL},i}\left(r_i|_{\eta_{k,i}=y_k}\right)\right\} \leqslant \mathcal{C}_{{\rm BL},i}\left(r_i|_{\eta=\lambda_k x_k+(1-\lambda)y}\right)$.

$\Rightarrow$ $\mathcal{C}_{{\rm BL},i}$ is quasi-concave in $\eta_{k,i}$, where $k=1,2$ and $0<\eta_{k,i}\leq\rho_k^2$.

Appendix C
Proof of Proposition 2

According to the proof of Corollary 1, r_i, $i=1,2,...,+\infty$ is monotonically increasing in $\eta_k, k=1,2$.

$\Rightarrow \forall\ x_{k,i} < y_{k,i}$, $i=1,2,...,+\infty$, $x_{k,i}, y_{k,i} \in (0,\rho_k]$ and $\lambda_{k,i} \in [0,1]$, we have $r_i|_{\eta_k=x_{k,i}} < r_i|_{\eta_k=\lambda_{k,i}x_{k,i}+(1-\lambda_{k,i})y_{k,i}} < r_i|_{\eta_k=y_{k,i}}$, where $k=1,2$.

As shown in Proposition 1, $\mathcal{C}_{\rm FBL}$ is concave in r_i. Hence, $\mathcal{C}_{\rm FBL} = \sum\limits_i \mathcal{C}_{\rm FBL}$ is concave in $\mathbf{r}=(r_1,r_2,...,r_i,...)$.

$\Rightarrow \min\left\{\sum\limits_i\mathcal{C}_{\rm FBL}\left(r_i|_{\eta_k=x_{k,i}}\right),\sum\limits_i\mathcal{C}_{\rm FBL}\left(r_i|_{\eta_k=y_{k,i}}\right)\right\} \leq \sum\limits_i\mathcal{C}_{\rm FBL}\left(r_i|_{\eta_k=\lambda_{k,i}x_{k,i}+(1-\lambda_{k,i})y_{k,i}}\right)$.

$\Rightarrow \mathcal{C}_{\rm BL}$ is quasi-concave in η_k, where $0<\eta_k\leq\rho_k^2$ and $k=1,2$.

References

[1] J. Laneman, D. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–80, Dec. 2004.

[2] S. Karmakar and M. Varanasi, "The diversity-multiplexing tradeoff of the dynamic decode-and-forward protocol on a MIMO half-duplex relay channel," *IEEE Trans. Inf. Theory*, vol. 57, no. 10, pp. 6569–90, Oct. 2011.

[3] W. Song *et al.*, "Distributed opportunistic two-hop relaying with backoff-based contention among spatially random relays," *IEEE Trans. Veh. Technol.*, vol. 64, no. 5, pp. 2023-36, May 2015.

[4] Y. Yang *et al.*, "Relay technologies for WiMax and LTE-advanced mobile systems," *IEEE Commun. Mag.*, vol. 47, no. 10, pp. 100-05, Oct. 2009.

[5] Y. Hu and L. Qiu, "A novel multiple relay selection strategy for LTE-advanced relay systems," in *IEEE VTC-Spring*, Budapest, Hungary, May 2011.

[6] Y. Hu, J. Gross and A. Schmeink, "QoS-Constrained energy efficiency of cooperative ARQ in multiple DF relay systems," *IEEE Trans. Veh. Technol.*, vol. 65, no. 2, pp.848–59, Feb. 2016

[7] M. Bhatnagar, "On the capacity of decode-and-forward relaying over Rician fading channels," *IEEE Commun. Lett.*, vol. 17, no. 6, pp. 1100–03, Jun. 2013.

[8] Y. Polyanskiy, H. Poor, and S. Verdu, "Channel coding rate in the finite blocklength regime," *IEEE Trans. Inf. Theory*, vol. 56, no. 5, pp. 2307–59, May 2010.

[9] Y. Polyanskiy, H. Poor, and S. Verdu, "Dispersion of the Gilbert-Elliott channel," *IEEE Trans. Inf. Theory*, vol. 57, no. 4, pp. 1829–48, Apr. 2011.

[10] W. Yang *et al.*,"Quasi-static multiple-antenna fading channels at finite blocklength," *IEEE Trans. Inf. Theory*, vol. 60, no. 7, Jul. 2014.

[11] G. Ozcan and M. C. Gursoy, "Throughput of cognitive radio systems with finite blocklength codes," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 11, pp. 2541–54, Nov. 2013.

[12] P. Wu and N. Jindal, “Coding versus ARQ in fading channels: How reliable should the phy be?” *IEEE Trans. Commun.*, vol. 59, no. 12, pp. 3363–74, Dec. 2011.

[13] B. Makki, T. Svensson, and M. Zorzi “Finite block-length analysis of spectrum sharing networks using rate adaptation,” *IEEE Trans. Commun.*, vol. 63, no. 8, pp. 2823–35, Aug. 2015.

[14] S. Xu *et al.*,, "Energy-efficient packet scheduling with finite blocklength codes: Convexity analysis and efficient algorithms," *IEEE Trans. Wireless. Commn.*, vol.15, no.8, pp.5527–40, Aug. 2016.

[15] O. L. A. Lopez *et al.*, "Ultrareliable Short-Packet Communications With Wireless Energy Transfer," *IEEE Signal Processing Letters*, vol. 24, no. 4, pp. 387–91, April 2017.

[16] B. Makki, T. Svensson and M. Zorzi, "Wireless Energy and Information Transmission Using Feedback: Infinite and Finite Block-Length Analysis," *IEEE Trans. Commun.*, vol. 64, no. 12, pp. 5304–18, Dec. 2016.

[17] L. Dickstein *et al.*, "Finite block length coding for low-latency high-reliability wireless communication," *Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, Monticello, IL, 2016, pp. 908–15

[18] Y. Hu, J. Gross and A. Schmeink, “On the capacity of relaying with finite blocklength,” *IEEE Trans. Veh. Technol.*, vol. 65, no. 3, pp. 1790–94, Mar. 2016.

[19] Y. Hu, J. Gross and A. Schmeink, “On the performance advantage of relaying under the finite blocklength regime,” *IEEE Commn. Lett.*, vol. 19, no. 5, pp. 779–82, May 2015.

[20] Y. Hu, A. Schmeink and J. Gross “Blocklength-limited performance of relaying under quasi-static Rayleigh channels,” *IEEE Trans. Wireless. Commn.*, vol. 15, no. 7, pp. 4548–58, Jul. 2016.

[21] Y. Hu *et al.*,“Finite Blocklength Performance of Cooperative Multi-Terminal Wireless Industrial Networks," ’ *IEEE Trans. Veh. Technol.*, vol. PP, no. 99, pp. 1-1.

[22] P. Nouri, H. Alves and M. Latva-aho, "On the performance of ultra-reliable decode and forward relaying under the finite blocklength," *IEEE EuCNC*, Oulu, June 2017.

[23] Y. Li, M. C. Gursoy and S. Velipasalar, "Throughput of two-hop wireless channels with queueing constraints and finite blocklength codes," *IEEE ISIT*, Barcelona, Spain, Jul. 2016.

[24] A. Hyadi, M. Benjillali and M. S. Alouini, "Outage performance of decode-and-forward in two-way relaying with outdated CSI,"*IEEE Trans. Veh. Technol.*, vol. 64, no. 12, pp. 5940–47, Dec. 2015.

[25] W. Jiang, T. Kaiser and A. J. H. Vinck, "A robust opportunistic relaying strategy for co-operative wireless communications," *IEEE Trans. Wireless. Commn.*, vol. 15, no. 4, pp. 2642–55, Apr. 2016.

[26] D. S. Michalopoulos, J. Ng and R. Schober, "Optimal relay selection for outdated CSI," in IEEE Communications Letters, vol. 17, no. 3, pp. 503–06, Mar. 2013.

[27] L. Fe *et al.*,"Outage-optimal relay strategy under outdated channel state information in decode-and-forward cooperative communication systems," *IET Commn.*, vol. 9, no. 4, pp. 441–50, Mar. 2015.

[28] J. Vicario and C. Antón-Haro, “Analytical assessment of multi-user vs. spatial diversity trade-offs with delayed channel state information," *IEEE Commun. Lett.*, vol. 10, no. 8, pp. 588–90, Aug. 2006.

[29] R. Mallik, “On multivariate Rayleigh and exponential distributions,” *IEEE Trans. Inf. Theory*, vol. 49, no. 6, pp. 1499–15, Jun. 2003.

[30] G. Arfken, H. Weber,"Mathematical Methods for Physicists," *Academic Press*, 6th edition, Jul. 2005.

[31] A. F. Molisch, “Wireless Communications,” *IEEE Press - Wiley*, 2011.

Yulin Hu received his M.Sc.E.E degree from USTC, China, in 2011. He successfully defended his dissertation of a joint Ph.D. program supervised by Prof. Anke Schmeink at RWTH Aachen University and Prof. James Gross at KTH Royal Institute of Technology in Dec. 2015 and received his Ph.D.E.E. degree (with honors) from RWTH Aachen University where he is a research fellow since 2016. From May to July 2017, he was a visiting scholar with Prof. M. Cenk Gursoy in Syracuse University, USA. His research interests are in information theory, optimal design of wireless communication systems. He has been invited to contribute submissions to multiple conferences. He received the Best Paper Awards at IEEE ISWCS 2017 and IEEE PIMRC 2017, respectively. He is currently serving as an editor for Physical Communication (Elsevier).

Anke Schmeink received the Diploma degree in mathematics with a minor in medicine and the Ph.D. degree in electrical engineering and information technology from RWTH Aachen University, Germany, in 2002 and 2006, respectively. She worked as a research scientist for Philips Research before joining RWTH Aachen University in 2008 where she is an associate professor since 2012. She spent several research visits with the University of Melbourne, and with the University of York. Anke Schmeink is a member of the Young Academy at the North Rhine-Westphalia Academy of Science. Her research interests are in information theory, systematic design of communication systems and bioinspired signal processing.

James Gross is an Associate Professor with the Electrical Engineering School of KTH Stockholm since November 2012. His research interests are in the areas of machine-to-machine communications, algorithms and protocols for wireless networks, and performance evaluation methods. Prior to joining KTH, he was assistant professor and head of the Mobile Network Performance Group at RWTH Aachen University from 2008 - 2012 and member of the DFG-funded UMIC research centre of RWTH. James studied electrical engineering from 1996 to 2001 at TU Berlin and UC San Diego. He received his PhD from TU Berlin in 2006 and worked at TU Berlin as Post-doc in 2007. He has published about 100 (peer-reviewed) papers in international journals and conference papers. His work has been awarded multiple times, among them the best paper awards at IEEE WoWMoM 2009 and at European Wireless 2009.