Distributed TV Spectrum Allocation for Cognitive Cellular Network under Game Theoretical Framework

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Abstract—**This paper proposes efficient schemes for wireless cellular base stations to utilize TV white spectrum (such base stations referred to as white base stations - WBS), so that WBSes can provide good services to their end terminals without violating the incumbent TV services. In particular, we consider two complementary problems. On the one hand, given a set of TV stations and white base stations, maximum permitted transmit power levels on all channels for each base stations needs to be determined. By use of convex programming, we propose here an improved, centralized mechanism. On the other hand, once the maximum transmit powers are determined, each white base station needs to choose a channel with the maximal permitted power on that channel such that the resulting cell performance is improved. Allocating channels with nonidentical transmission power and asymmetric interference is formulated into congestion game for the first time, and an algorithm is derived thereafter which converges after a small number of iterations in simulation. However, the scheme requires geo information coupled with a radio map to decide at each WBS in a decentralized manner about the channel usage. We find that in comparison to several other decentralized schemes, our proposed approach first of all converges after a small number of iterations while on the other hand it is able to achieve the same network performance spending significantly lower transmit power.**

Keywords: cognitive mesh network, TV whitespace, congestion game, channel assignment, SINR, economic power consumption [1]

I. INTRODUCTION

Opportunistic utilization for secondary users working with TV broadcast spectrum (TV white space) is promising to cope with the scarcity of spectrum resources [2]. Firstly, more unused TV white frequencies become vacant than ever with the ongoing transition from analog to digital broadcasts. Secondly, the lower frequencies of TV band enable broadband access over much longer ranges compared to other bands with higher center frequencies. Nevertheless, services on TV receivers need to be protected with so called interference margin[2] [15] which must not be exceeded jointly by all secondary users working on the channel. [3]

Federal Communications Commission (FCC) of U.S. and Electronic Communications Committee (ECC) in Europe have announced rules on the transmission power of white space for secondary users in US and Europe respectively [2], [3]. FCC adopts a minimum distance between secondary user and TV service area to guarantee that the interference margin is not exceeded by secondary users. The transmission power for fixed secondary users is fixed to 4 W which is a conservative setting. FCC assumes that the protection area is sufficient to protect the TV receivers, but it is not the case when there are multiple secondary equipments transmitting in the the same as is discussed in [13]. ECC's restriction requires that the secondary user adapt its transmission power in order not to violate the interference margin at exposed TV receivers. In this manner, secondary systems have to determine their maximum transmission power.

Recent work [17] follows FCC rules [1] to obviate spectrum sensing and only rely on the database of TV incumbents to determine the white space availability on secondary users. The authors of [17] demonstrate the feasibility of predicting the available TV spectrum accurately using suitable propagation models (Longley-Rice and terrain wherein). A central controller contains the locations of all TV stations and secondary users, then the central controller calculates the RSSI level of TV UHF signals on all secondary users and accordingly determines the available TV spectrum for them. The authors give big impetus to the database method by developing sophisticated signal propagation modeling and efficient content dissemination scheme. Enlightened by this work, it can be seen that the RSSI level caused by secondary users on TV receivers can be calculated accurately in a centralized entity if secondary users' transmission power, geo-location and appropriate propagation model are provided. Inversely, given geo-location and appropriate propagation model, secondary users' maximum transmission power can be determined by the central entity according to the interference margin (maximum RSSI level from secondary users) at TV receivers.

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[2] interference margin is the maximal interference caused by secondary users, which doesn't violate TV service.

[3] In this paper, channel and spectrum are used indiscriminately.

In this paper, the secondary users are assumed to be cellular systems consisting of base stations and associated terminals, all of which work on TV white spectrum. The corresponding secondary base stations are referred to as white base stations (WBS). In what follows in this paper, we use WBS and secondary base station interchangeably. Some cellular networks, such as GSM or LTE network, work on licensed spectrum, and they emphasis on providing satisfactory services to their end terminals by choosing proper transmission channel and power. As to cellular network working on TV white spectrum, they have to keep one eye on the primary users to make sure that TV service is not violated, which makes the problem of channel and power selection even harder. It is possible that WBSes are owned and operated by different operators, thus completely centralized decision on the base stations' working channel and transmission power is infeasible. In this paper we will investigate how do secondary cellular network utilize the whitespace.

we will look for a distributed scheme to solve this problem.

The rest of the paper is organized as follows. We elucidate the system model in Section II, afterwards related work and problem formulation is presented in Section III. In Section IV, we discuss how to utilize the white space sufficiently by setting the transmit powers based on a convex problem formulation. We analyze the spectrum allocation problem under game theoretical framework and propose an algorithm in Section V, thereafter performance evaluation is presented in Section VI. Finally, we conclude our work and point out directions of future research in Section VII.

II. SYSTEM MODEL

Following the IEEE 802.22 standard, the primary systems considered in this paper are digital TV (DTV) stations which use the TV spectrum legally. TV stations provide service to passive TV receivers which must not be interfered by secondary systems. The secondary systems are IEEE 802.22 Wireless Regional Area Network base stations (WBS) utilizing the TV spectrum with senseless mode [17]. WBSes serve a set of end users/terminals without interfering TV receivers significantly. Denote the set of DTV stations by $\mathcal{K}$ and the set of WBSes by $\mathcal{N}$ with $|\mathcal{N}| = N$. Channels are available for both TV services and secondary systems, in total is represented by $\mathcal{C}$ with $|\mathcal{C}| = C$, and the channels are considered to be identical. These secondary systems are distributed over a certain area and is surrounded by multiple DTV service areas, as Figure (1) shows. When there are two WBSes working on the same channel, co-channel interference will be caused to each other, while, neighboring channel interference is not considered. Each DTV station, as well as each WBS utilizes exactly one channel.[4] We represent the usage of channels for WBS i with a binary vector $X_i^{|\mathcal{C}|\times 1} = \{\cdots, x_{ik}, \cdots\} \in \{0,1\}^{|\mathcal{C}|}$, where $k \in \mathcal{C}$ and binary variable x_{ik} denotes whether channel k is used by user i. As each node can only use one channel, for X_i there is $\sum_{k=1}^{|\mathcal{C}|} x_{ik} = 1$. Let c_i denote the channel used by a WBS $i \in \mathcal{N}$. The transmission power of WBS i on channel c is P_i^c.

There are TV service contours deployed at the edge of the TV service area (as bold rectangles in Figure (1)) representing the worst located TV receivers. For them a certain upper bound of interference should not be violated to guarantee the TV services, where the interference is from secondary users and noise. The deployment of contours is decided by the TV operators, which varies according to the concrete location, geographic terrain and possible deployment of secondary networks. For simplicity, we assume there is only one contour deployed for one TV area.

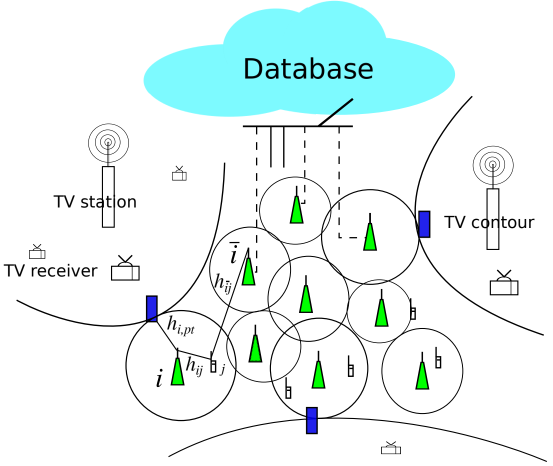

Fig. 1. System model: WBS cells and DTV systems

WBSes are interested in payload data exchange with their associated terminals with good quality of services (QoS). As to performance metric for this QoS provisioning, we choose the signal-to-noise-and-interference ratio (SINR) at the terminals. SINR is the ratio between the power of the received signal of interest and the summed power of all interference sources as experienced by the terminal. We only focus on the downlink SINR. We denote the path loss between the serving WBS i and a certain terminal j associated to it by h_{ij}, similarly, the path loss between any other secondary base station $\bar{i} \neq i$ operating on the same channel as i and end user j is denoted as $h_{\bar{i}j}$. The path loss is dependent on the distance between the corresponding equipments, e.g. $h_{ij} = K \cdot d_{ij}^{-\alpha}$, where α is the path loss exponent and K is a constant that models the reference loss over a single unit of distance. Furthermore, N_0 denotes the noise power. Finally, we do consider shadowing, but do not consider fading. Hence, the sum of all disturbing RF effects (including interference) at terminal j (we assume the working channel is c) is given by

$$f_j^c = \sum_{\bar{i}} (P_{\bar{i}}^c \cdot h_{\bar{i}j} \cdot z_{\bar{i}j}) + N_0 \qquad (1)$$

where $P_{\bar{i}}^c$ denotes the transmit power of WBS $\bar{i}$ and $z_{\bar{i}j}$ models the zero-mean log-normally distributed shadow-fading with

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[4] The assumption that one WBS only utilizes one channel is for convenience of analysis. In reality multiple channel usage (channel bonding) is requisite as one single TV channel's bandwidth is 6 MHz which is not adequate for a WBS to fulfill system requirement. We will relax this single channel usage assumption without hammering our scheme in the end of Section (V).

standard deviation σ_{SH} between $\bar{i}$ and j. Hence, the signal-to-interference-and-noise ratio (SINR) on end terminal j is given by:

$$\gamma_j = \frac{P_i^c \cdot h_{ij} \cdot z_{\bar{i}j}}{f_j^c} \tag{2}$$

To manage secondary channel access, there is a central database recording the location and terrain information of the whole secondary network. Besides, the channel usage of each WBS is also stored in the database. We assume all secondary systems can access the central database directly to obtain the geo-location information and the channel usage of all WBSes. WBSes work on senseless mode, and is able to calculate the RSSI from one transmitter to an receiver with proper propagation model (e.g. Formula (1) and (2) can be calculated within WBS) with the geo-location and channel usage information. The geo-location information in the secondary networks is deemed to be static. We assume the secondary base stations are not under the same operators, thus there is no scheduling mechanism available among WBSes.

III. RELATED WORK AND PROBLEM FORMULATION

Given all the other WBSes' channel and power selection in secondary network, to achieve high SINR on its end terminals (or QuasiSINR), the pure strategy of one WBS is to choose the channel experiencing the minimum interference, and utilizing the biggest possible transmit power (make sure the primary services are not violated) in order to achieve better SINR at its terminals and meanwhile maximize their coverage. Nevertheless, high transmission power causes significant co-channel interference to other secondary cells operating on the same channel, and the WBSes' distributed update will end in a miserable Nash equilibrium, if there existsf one [24]. Our goal in this paper is to propose a strategy for WBSes to choose channel and power, so that they can deviate from the Nash equilibrium with big Price of anarchy in distributed manner. To speak exactly, the proposed scheme should protect the primary systems, provide its end users good SINR with relatively smaller power consumption, and as a distributed scheme, it should converge quickly.

A centralized scheme is proposed in [18] for joint channel and power allocation among end terminals in OFDM cognitive radio network. [11] discusses power control and channel assignment in both down-link and up-link communication in cellular network. Although the solution is distributed, primary users are required to cooperate with secondary base station in a learning process to decide the transmission power, in addition, there is only one secondary base station considered whereas we are coping with the whole cellular network. A distributed power allocation (single channel) scheme based on learning for secondary networks is given in [8], where penalty function involving the interference threshold on primary systems is used. [24] deals with the joint channel-power selection for multiple transmission links (pairs). The authors decompose the Lagrangian dual of the problem, then propose a distributed scheme based on the dual parameters. The scheme converges to pure Nash equilibrium, but in order to facilitate this scheme, monitors are required to watch interference from secondary users, moreover, monitors have to be equipped with computational ability and interact with secondary users in the whole process of convergence.

Because of the requirement by the interference margin from the primary system, the primary systems are involved in all the current distributed schemes, which is uneconomic and unrealistic in reality. We try to propose a distributed workaround for the joint power and channel allocation problem, so that the dynamics in the secondary network can be seen as transparent for primary system. In this paper we solve this problem through two subproblems: firstly, given a set of secondary WBS and their geo-location, the maximum permitted transmit power on all channel for each WBS is determined, so that the interference margin is impossible to be broken no matter how do WBSes utilize the channel and power resources. This requires considering the joint interference that the WBS have on the TV receivers of the considered service area. Secondarily, once the maximal transmit power has been determined, each WBS chooses its operating channel. While for the first problem a centralized approach is of interest, the second problem should be solved by a distributed scheme in general. We discuss the detailed problems in the following two subsections in combination with related work in the respective area.

A. Maximal Transmission Power Planning

[15] gives a sufficient condition for secondary base stations not to violate the services on TV contours, which requires the aggregate interference caused by WBSes on TV contours to be lower than interference margin. The sufficient conditions in the context of TV white space is formulated into a centralized linear programming program (LP). The objective function is to maximize the summation of all secondary base stations' transmission power, and the constraints are built to satisfy the sufficient condition for each TV contour.

B. Channel Allocation with Fixed Transmission Power Level

After knowing the power limit on each channel, WBSes need to decide which channel to use so as to mitigate interference among WBSes and provide better SINR to its end users. In this paper we assume WBSes' transmission power is the biggest permitted and fixed. Such problem lies in *channel assignment problem* which has been well investigated in many scenarios. Channel assignment problem mainly copes with mitigating co-channel interference among users, which can be converted into coloring problem thus is NP hard [20]. Authors of [6], [14] propose heuristic algorithms utilizing best response based on the welfare on itself to assign channels among users, but the assumption that transmission power is identical and path loss is symmetric renders them problematic for our problem where transmission is nonidentical and the path loss is asymmetric (e.g. shadowing exists, as in our system model). Distributed algorithm based on Learning is proposed in [12] for LTE to allocate the resource block in down link, which leads to correlated equilibrium, but large number of steps hinter its application. [19] formulates channel assignment problem in ad-hoc cognitive radio network into potential game which leads to pure NE, a learning scheme achieving slightly better performance is provided for comparison, but they assume the transmission power is identical and there is no noise in the secondary network, and the proposed random access mechanism demands a huge amount of information to be exchanged, which is a real burden for network in ad-hoc structure. [7], [23] investigate the channel allocation problem under game framework in same collision domain, the authors propose algorithms to converge to pure Nash equilibrium (NE) and strongly dominate strategy equilibrium respectively. As to our knowledge, there is no work dealing with channel allocation with such asymmetric interactions.

In this paper we try to improve the SINR on secondary end terminals through WBSes' power-channel strategy. To facilitate analysis and proposition of solution, we propose a metric *QuasiSINR* for each secondary base station to represent the SINR of the terminals in the coverage of that base station.

C. *QuasiSINR*

We are interested in a 'worst-case SINR' of each cell. Such a virtual value will be referred to as *QuasiSINR* in the following. It is proposed to represent the worst-case quality of service that a WBS might provide to its associated terminals.

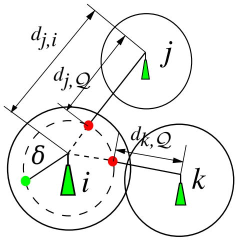

Fig. 2. QuasiSINR of WBS i: the worst case SINR of its terminals

For this we consider a circle around an WBS with radius δ as is shown in Figure (2). The radius δ is the largest distance among all associated terminals of the considered WBS. δ can be different for different WBSes and can also be decided by operators. For this terminal furtherest away on the δ-circle, we now construct a worst-case SINR which factors in all interference from neighboring secondary cells as if they were closest to the considered terminal. Hence, QuasiSINR is the ratio between the weakest signal of interest and the summation of the biggest (possible) interference from other co-channel WBSes. According to this construction, the weakest strength of the signal of interest is $P_i^c \cdot h_{iQ} \cdot z_{iQ} = P_i^c \cdot \delta^{-\alpha} \cdot z_{iQ}$ while the biggest possible interfering power from co-channel WBS j is $P_j^c \cdot h_{jQ} \cdot z_{jQ} = P_j^c \cdot (d_{ji} - \delta)^{-\alpha} \cdot z_{jQ}$. We denote in this context by Q the point along the $\delta - circle$, where the weakest interested signal and strongest interference happen. Hence, as we form the SINR such a virtual 'worst-case' terminal, the co-channel interference impact is overestimated as the total received interference power is given by the sum $\sum_{\forall j} P_j^c \cdot h_{jQ} \cdot z_{jQ}$ where index j spans all co-channel WBSes with i. Formally, the QuasiSINR of WBS i is given by:

$$\begin{aligned}\tilde{\gamma}_i &= \frac{P_i^c \cdot h_{iQ} \cdot z_{jQ}}{\sum_{\substack{j\neq i, j\in\mathcal{N}\\ c_j=c_i}}(P_j^c \cdot h_{jQ} \cdot z_{jQ}) + N_0} \\ &= \frac{P_i^c \cdot \delta^{-\alpha} \cdot z_{jQ}}{\sum_{j\neq i, j\in\mathcal{N}}(P_j^c \cdot K(d_{ji}-\delta)^{-\alpha} \cdot z_{jQ}) + N_0} = \frac{\tilde{P_i}^c}{\tilde{f_i}^c}\end{aligned} \tag{3}$$

where $\tilde{P_i}^c$ represents the power of received interested signal from WBS i on Q (the green square in Figure (2)), and $\tilde{f_i}^c$ denotes the sum of received co-channel interferences plus noise, where the co-channel interference happens on the red rounds in Figure (2)).

Notice regrading the QuasiSINR, that any modification of the transmit powers of co-channel interference sources (i.e. other WBS working on channel c) will have always fixed impact to the WBS concerned, so the interaction between co-channel WBSes are independent on the concrete end terminals. With QuasiSINR, the channel and power allocation problem will exclude terminals and thus simplify the problem. QuasiSINR will be validated in Section (VI).

D. *Problem to be Solved in This Paper*

The problem is represented in the following form, to ensure fairness, instead of maximizing the sum of QuasiSINR of all WBSes, we will try to minimize the sum of inversed QuasiSINR.

$$\begin{aligned}\text{Minimize} \quad & \sum_{i\in\mathcal{N}} \frac{1}{\tilde{\gamma}_i} \\ \text{subject to} \quad & \sum_{k=1}^{|\mathcal{C}|} x_{ik} = 1\end{aligned} \tag{4}$$

For every WBS, each channel in $\mathcal{C}$ experiences different levels of interference from other WBSes working on it. In order to provide better service to its end users, WBS is liable to choose either the channel permitting higher transmission power or the one with less interference, or the channel compromising the two factors according to Formula (2). Achieving optimal white spectrum allocation in a distributed style is the goal of this work, furthermore, this distributed solution should converge fast and lead to an efficient and stable solution. In the following We will present our solution by solving the two subproblems sequentially.

IV. DECIDE THE MAXIMAL PERMITTED TRANSMISSION POWER

We adopt the interference model and the optimization methodology from the work of [15] to plan the maximal transmission power for WBSes. In our system the WBSes locate within one area, whereas TV areas locate around them. If implying linear programming to decide the maximal transmission power, the WBSes locating far from TV contours contribute more to the sum of power with the biggest permitted power, as a result the maximal transmission power on each channel obtained with LP is seriously unbalanced. To address this fairness issue, we try to maximize the summation of the logarithmic value of every WBS's transmission power, then we formulate the problem into a series of convex optimization problems, each of which corresponds to a optimization. We denote that, for WBS $i \in \mathcal{N}$, the maximal transmission power allowed to be used on channel c is denoted as P_i^c. For each channel $c \in \mathcal{C}$, there is one optimization problem,

$$\begin{aligned} \max \quad & \sum_{i \in \mathcal{N}} log(P_i^c) \\ \text{s.t.} \quad & \sum_{i \in \mathcal{N}} (P_i^c \cdot h_{i,pt} \cdot z_{i,pt}) < I_{pt}^c, \end{aligned} \tag{5}$$

where I_{pt}^c is the interference margin for the DTV contour pt and the DTV is working on channel c. Here we only consider the interference caused by WBSes, Since their transmission power is higher and their altitude is higher [15], the downlink transmission contributes the main secondary interference [5], and the interference caused by white space end users is trivial and omitted. There will be multiple constraints for Optimization (5) if there are multiple DTV contours working on channel c. There is one optimization problem for each channel $c \in \mathcal{C}$, after solving the $|\mathcal{C}|$ problems, we obtain the maximal transmission power over all channels (maximal power map) for every WBS. We solve this convex optimization problem with [9] in the centralized base station.

Figure (3) depicts the distribution of maximal transmission power levels obtained in 100 simulations. In each simulation the locations of TV contours are randomly decided around the WBSes. It can be seen that around half of WBSes' transmission power planed with LP is restricted to be the minimum transmission power, and the other half of WBSes' transmission power is the maximum permitted power. By applying convex programming, the planed maximal transmission power levels are distributed evenly in between the minimum and maximum permitted power. The gain of SINR on end terminals by applying convex optimization to decide the maximal transmission power is illustrated in the simulation section.

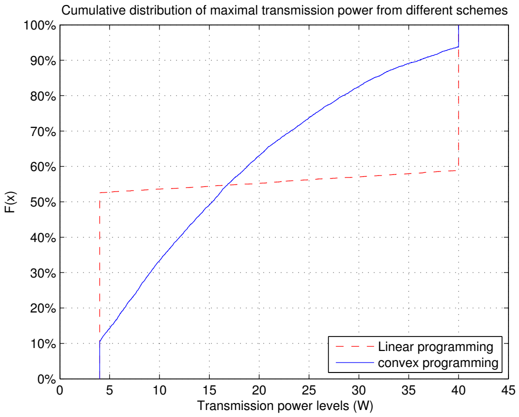

Fig. 3. Distribution of maximal transmission power levels obtained from convex and linear programming respectively

Optimization problem (5) provides the maximal transmission power, with which violating the TV contour becomes impossible even in case that all WBSes work on the same channel simultaneously. When there are WBSes working on a different channel from others, there will appear a interference margin for TV contour, which can tolerate network dynamics such as new WBS starting to work, increased background noise, or variance of channel shadowing.

V. CHANNEL ALLOCATION WITH FIXED TRANSMISSION POWER

A. Centralized Optimization Programming

In the very beginning, we formulate the channel allocation problem into a binary quadratic programming problem which will be solved in a centralized way. For two nodes i and j, there is,

$$X_i^T X_j = \sum_{k=1}^{|\mathcal{C}|} x_{ik} \cdot x_{jk} = \begin{cases} 1 & \text{if } c_i = c_j \\ 0 & \text{if } c_i \neq c_j \end{cases} \tag{6}$$

The power levels across all channels for WBS i are denoted by a constant vector $P_i \in P^{|\mathcal{C}| \times 1}$, which is possibly nonidentical to other nodes' power levels. The power used by user i is $P_i^T X_i = \sum_{k=1}^{|\mathcal{C}|} P_i^k \cdot x_{ik}$.

Problem (4) can be modeled via general purpose nonlinear optimization:

$$\begin{aligned} \min \quad & \sum_{i=1}^{N} \frac{\sum_{j \in \mathcal{N}, j \neq i} P_j^T X_j (X_j^T X_i) h_{jQ} z_{jQ} + N_0}{P_i^T X_i h_{iQ}} \\ \text{s.t.} \quad & \sum_{k=1}^{|\mathcal{C}|} x_{ik} = 1, x_{ik} \in X_i \in \{0,1\}^{|\mathcal{C}|} \end{aligned} \tag{7}$$

x_{ik} with $i \in \mathcal{N}, k = 1, 2 \cdots$ is binary variable. Problem (7) is a non-linear problem with binary variables, but it can be reformulated into a quadratic programming problem as,

$$\begin{aligned} & \min \\ & \sum_{i=1}^{n} \Big(\sum_{\substack{j \in \mathcal{N}, \\ j \neq i}} \sum_{k} \frac{P_j^k}{P_i^k h_{iQ}} \cdot h_{jQ} \cdot z_{ji} \cdot x_{jk} \cdot x_{ik} + \sum_{k} \frac{N_0}{P_i^k h_{iQ}} \cdot x_{ik} \Big) \\ & \text{s.t.} \sum_{k=1}^{|\mathcal{C}|} x_{ik} = 1, x_{ik} \in X_i \in \{0,1\}^{|\mathcal{C}|} \end{aligned} \tag{8}$$

The reformulation is available in Appendix (A). We use LINDO [4] which is a state of art non-linear problem solver to solve the problem, which employs Branch-And-Reduce method to get the global optimal for the problem. The result will be used as benchmark.

B. Distributed White Space Channel Allocation Technology (WhiteCat): Algorithm and Protocol

In this paper a distributed scheme for WBSes to allocate channels is proposed, which is named as white space channel allocation technology (WhiteCat). WitheCat is depicted by

Algorithm (1) which is a best response process, where each WBS (referred to as i) greedily searches for a preferred channel based on utility function u_i, and the sum of all WBSes' utilities is minimized after finite times of updates, even the interaction between WBSes are asymmetric. The utility is as follows,

$$u_i = \frac{\sum\limits_{\substack{j\in\mathcal{N},j\neq i,\\ c(\sigma_j)=c(\sigma_i)}} \tilde{f_{ji}}}{2\cdot\tilde{P_i}} + \frac{1}{2}\sum_{\substack{j\in\mathcal{N},j\neq i,\\ c(\sigma_j)=c(\sigma_i)}} \frac{\tilde{f_{ij}}}{\tilde{P_j}} + \sum_{\substack{\mathcal{S}:i,j\in\mathcal{S},\\ c(\sigma_j)=c(\sigma_i)}} \frac{N_0}{C\cdot\tilde{P_i}} \quad (9)$$

where $\tilde{f_{ij}} = P_i \cdot h_{ij} \cdot z_{ij}$ and $\tilde{f_{ji}} = P_j \cdot h_{ij} \cdot z_{ji}$. Overlooking the constant coefficient 2, the first item of u_i is one part of the inversed QuasiSINR of station i. To minimize the first item, WBS i needs to choose a channel either permits larger transmission power or experiences less interference, whereas the larger power will increase the second item which is part of inversed QuasiSINR of other co-channel WBSes. Hence, the cost function presents a reasonable comprise between the welfare of one WBS and others. If WBS only emphasizes on its own utility (e.g. the first part of Formula (9)), the best response process doesn't converge. We have the following theorem:

Theorem 1. *With non-identical transmission power, if every WBS updates its channel based on algorithm* (1) *with utility based on its own interests, the process doesn't always converge.*

The proof is in Appendix (A).

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Input: quasi distance $d_{ij}$ for $\forall i,j\in\mathcal{N}$; path lose between
       $i$ and any other WBS $h_{ij}$, $j\in\mathcal{N}, j\neq i$, and the
       fading $z$ on it; noise $N_0$; total number of
       secondary base stations $N$; maximal transmission
       power $P_j^c, j\in\mathcal{N}, c\in\mathcal{C}$; $c_j$, current channel used
       by $j\in\mathcal{N}, j\neq i$.
for $i\in\mathcal{N}$ do
    for $c\in\mathcal{C}$ do
        calculate $u_i(c)$ based on Formula (9)
        if $u_i(c) < u_i(c_i)$ then
            $c_i \leftarrow c$
        else
            $c_i$ unchanged
        end
    end
    Notify data base of its channel usage, which notifies
    the other WBSes
end
```

Algorithm 1: Spectrum selection for node i

c_i is the current channel used by $i\in\mathcal{N}$. Imitating the player's behavior in the congestion game, each base station tries to find the channel $c\in\mathcal{C}$ that brings the smallest u_i based on the other stations' decisions, every channel update will decrease the summation of utilities in the whole network and finally converges to a pure Nash equilibrium(proof is in Section (V-C).

Some parameters needed to calculate the utility are identical for all WBSes, such as quasi distance δ, the total number of WBSes N, number of channels C, attenuation factor α, standard deviation σ in flat shadowing and noise N_0, albeit the following information is further needed to calculate u_i:

- $\sum_{\substack{j\in\mathcal{N},j\neq i,\\ c(\sigma_j)=c(\sigma_i)}} \tilde{f_{ji}^c}, c\in\mathcal{C}$: sum of the received interferences which happen on the Q point (introduced in Section III-C) for WBS i, which is the intersection point of $\delta - circle$ and the connecting line between WBS i and its interfering sources.
- $\tilde{f_{ij}^c}$: the interference caused by i on the nearest point which locates along j's $\delta - circle$, there is $c_j = c_i$.
- P_j^c: transmission power of j for using channel c.

Unfortunately, it is difficult to get these interferences of interested measured, for station i, it is low efficient to scan all channels and obtain the interferences f_{ji} on virtual measurement point for each channel, furthermore, it is impossible to split the interference f_{ij} from the total interference received on WBS j' Q point.

Enlightened by the work of [17] which verifies the usage of geo information in deciding the available channels, we let every WBS store the location information and maximal power map of all other WBSes, and it retrieves information about channel usage by other WBSes from centralized base station. After executing Algorithm (1), it reports to centralized base station for its channel update. As the location of WBSes and TV stations and the transmission channel and power of TV stations are generally static (entries of TV station change averagely once in 2 days [17]), except for the channel usage in the network, the change of the other data stored in WBS is infrequent.

We refer [19] to decide the sequence for WBSes to update their channel. [19] proposes a method like random access mechanism of CSMA/CA, where the access for broadcast medium is changed to getting access to the centralized center to retrieve the current channel usage and update its new channel. All WBS are able to access the database in one round (with random or Predetermined sequence). As WBSes are connected with database, the control messages needed to decide the sequence will not become a burden. Update of channels can happen in the boot phase, or when the quality of services (the SINR on its end users) of WBSes falls below a threshold, or a fixed time duration comes to end, or a new WBS joins in the network.

C. Analysis in Game Theoretical Framework

We give an elegant proof on WhiteCat's convergence in the framework of congestion game theory. Formulating a spectrum sharing problem into a congestion game and the concept of *virtual resources* are firstly proposed in [16] which 'reversely engineer' the distributed channel allocation schemes proposed in [6], [14].

1) Congestion Game: A congestion game [21] [22] can be expressed by a tuple $\lambda = (\mathcal{N}, \mathcal{R}, (\sum_i)_{i\in\mathcal{N}}, (g_r)_{r\in\mathcal{R}})$, where $\mathcal{N} = \{1,\ldots,N\}$ denotes the set of players (each each is labeled with a unique index number), $\mathcal{R} = \{1,\ldots,m\}$ the set of resources, $\Sigma_{i\in\mathcal{N}} \subseteq 2^{\mathcal{R}}$ is the strategy space of player i. Under strategy profile $\sigma = (\sigma_1, \sigma_2, \cdots \sigma_N)$, player i chooses strategy $\sigma_i \in \Sigma_i$, and the total number of users using resource

r is $n_r(\sigma) = |\{i \mid r \in \sigma_i\}|$. The cost $g_r : \mathbb{N} \to \mathbb{Z}$ is a function of the number of users for resource r, $g_r^i = \sum_{r\in\sigma_i} g_r(n_r(\sigma))$. In our paper, g_r^i is regarded as *congestion* and is Monotonic.

Rosenthal's potential function $\phi : \sigma_1 \times \sigma_2 \times \cdots \times \sigma_n \to Z$ is defined as:

$$\begin{aligned} G(\sigma) &= \sum_{r\in\mathcal{R}} \sum_{i=1}^{n_r(\sigma)} g_r(i) \\ &= \sum_{i\in\mathcal{N}} \sum_{r\in\sigma_i} g_r(n_r^i(\sigma)) \end{aligned} \tag{10}$$

$n_r^i(\sigma)$ means the number of players using resource r and *their indices are smaller than or equal to* i. Note that the potential is *not* the sum of congestions experienced by every user. The change of the potential caused by one player's unilateral move from σ to σ' is equivalent to the change of gain (or loss) of that player.

$$\varDelta G(\sigma_i \to \sigma_i') = g^i(\sigma_i', \sigma_{-i}) - g^i(\sigma_i, \sigma_{-i}) \tag{11}$$

σ_{-i} is the strategy profile for all players except for i. As every congestion game is a potential game, and the total potential is finite, thus the number of improvements is upper-bounded by $2 \cdot \sum_{r\in\mathcal{R}} \sum_{i=1}^{n_r(\sigma)} g_r(i)$ [22].

In congestion game, each player acts selfishly and aims at choosing strategy $\sigma_i \in \Sigma_i$ to minimize their individual cost. The gain (loss) caused by any player's unilateral move is exactly the same as the gain (loss) in the potential, which may be viewed as a global objective function. For problems where the potential of the problem is the same with the summation of the cost of all users, the cost function can be used as a utility function directly. This equivalence doesn't exist in our problem, but by carefully choosing the cost function for players, we can make sure that the change of individuals' cost is in the same direction with that of the global utility.

2) Bridging the Game and Practical Scheme: We utilize the conception of virtual resource which is firstly introduced in [16]. In the following text, we use player and base station interchangeably.

- Player i' strategy space is $\Sigma_i = \{(i,j,c), j \in \mathcal{N}, j \neq i, c(\sigma_j) = c, c = 1, 2, \cdots, N\}$, and i has C admissible strategies, one strategy related with channel $c \in \mathcal{C}$ is described by the set of virtual resources it uses: $\sigma_i = \{(i,j,c), j \in \mathcal{N}, j \neq i, c(\sigma_j) = c\}$, note that virtual resource $(i,j,c) \neq (j,i,c)$.
- Under the strategy profile $\sigma = (\sigma_1, \sigma_2, \cdots \sigma_N)$, player i obtains a total cost of

$$g^i(\sigma) = \sum_{\substack{j\in\mathcal{N}, j\neq i,\\ c=c(\sigma_i)=c(\sigma_j)}} (g_{(i,j,c)}(n_{(i,j,c)}(\sigma)) + g_{(j,i,c)}(n_{(j,i,c)}(\sigma)) \tag{12}$$

The transmission power over all channels of player i is $\{p_{i1}, p_{i2}, \cdots, p_{i|\mathcal{C}|}\}$ and fixed. Path loss is assumed reciprocal: $h_{ij} = h_{ji}$, but nor is the flat fading z. To keep the formula clear in the following part, we denote $\tilde{f_{ij}} = P_i \cdot h_{ij} \cdot z$, $\tilde{f_{ji}} = P_j \cdot h_{ij} \cdot z$, $\tilde{P_i} = h_{iQ}$ for $i \in \mathcal{N}$, where $h_{ji} = h_{ij} = (d_{ji} - e)^{-\alpha}$, $h_{ii} = h_{jj} = e^{-\alpha}$, d_{ji} is the distance between base station i and j, and δ is the quasi distance introduced in Section (II). N_0 is noise which is identical for any channel and any WBS. We define the cost function for virtual recourses (i,j,c) as follows,

$$g_{(i,j,c)}(k) = \begin{cases} \dfrac{\tilde{f_{ji}}}{2\tilde{P_i}} + \dfrac{\tilde{f_{ij}}}{2\tilde{P_j}} + \dfrac{C \cdot N_0}{N \cdot \tilde{P_i}} & \text{if } k = 2 \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

As resource (i,j,c) only lies in the strategy space of player i and j, based on (13), cost of resource (i,j,c) is only decided by the number of players(0 or 2) using it, thus this is a typical congestion game which has infinite update property [22].

Substitute Formula (13) to Formula (12), we get the total cost for user i under strategy profile σ.

$$\begin{aligned} g^i(\sigma) &= \sum_{\substack{j\in\mathcal{N}, j\neq i,\\ c=c(\sigma_j)=c(\sigma_i)}} (g_{(i,j,c)}(2) + g_{(j,i,c)}(2)) \\ &= \sum_{\substack{j\in\mathcal{N}, j\neq i,\\ c(\sigma_j)=c(\sigma_i)}} \Big(\frac{\tilde{f_{ji}}}{\tilde{P_i}} + \frac{\tilde{f_{ij}}}{\tilde{P_j}} + \frac{C \cdot N_0}{N}\big(\frac{1}{\tilde{P_i}} + \frac{1}{\tilde{P_j}}\big)\Big) \\ &= \frac{\sum_{\substack{j\in\mathcal{N},\\ j\neq i,\\ c(\sigma_j)=c(\sigma_i)}} \tilde{f_{ji}}}{\tilde{P_i}} + \sum_{\substack{j\in\mathcal{N},\\ j\neq i,\\ c(\sigma_j)=c(\sigma_i)}} \frac{\tilde{f_{ij}}}{\tilde{P_j}} + \frac{CN_0}{N} \sum_{\substack{j\in\mathcal{N},\\ j\neq i,\\ c(\sigma_j)=c(\sigma_i)}} \big(\frac{1}{\tilde{P_i}} + \frac{1}{\tilde{P_j}}\big) \\ &= \frac{\sum_{\substack{j\in\mathcal{N}, j\neq i,\\ c(\sigma_j)=c(\sigma_i)}} \tilde{f_{ji}}}{\tilde{P_i}} + \sum_{\substack{j\in\mathcal{N}, j\neq i,\\ c(\sigma_j)=c(\sigma_i)}} \frac{\tilde{f_{ij}}}{\tilde{P_j}} + \frac{2CN_0}{N} \sum_{\substack{i\in\mathcal{S}\subset\mathcal{N},\\ \mathcal{S}:\forall i\in\mathcal{S}\\ c(\sigma_i)=c}} \frac{1}{\tilde{P_i}} \end{aligned} \tag{14}$$

Let $\mathcal{S}$ denote the set of WBSes which work on the same channel. Now we try to get the potential over all WBSes, note that the summation of one WBS's congestion is related to its index. For any two WBS $i, j \in \mathcal{S}$ with $i < j$, the potential brought in by i is 0, while, that caused by j is in the form of $g_{(i,j,c)}(2) + g_{(j,i,c)}(2)$. In other words, for each interfering pair of WBSes, only the WBS with bigger index contributes to the potential. The total potential is,

$$\begin{aligned} G(\sigma) &= \sum_{r\in\mathcal{R}} \sum_{i=1}^{n_r(\sigma)} g_r(i) = \sum_{i\in\mathcal{N}} \sum_{r\in\sigma_i} g_r(n_r^i(\sigma)) \\ &= \sum_{i\in\mathcal{N}} \frac{\sum_{\substack{j\in\mathcal{N}, j\neq i,\\ c(\sigma_j)=c(\sigma_i)}} \tilde{f_{ji}}}{\tilde{P_i}} + \frac{CN_0}{N} \sum_{\substack{\mathcal{S}\subset\mathcal{N},\\ \forall i\in\mathcal{S}, c(\sigma_i)=c}} \mid \mathcal{S} \mid \sum_{i\in\mathcal{S}} \frac{1}{\tilde{P_i}} \end{aligned} \tag{15}$$

When players minimize their utilities (cost or potential) illustrated by Formula (14), the total congestion in the secondary network given by Formula (15) decreases monotonically before reaching one Nash equilibrium. Players' greedy update in the game to minimize its cost Function (14), which ceases finally in pure Nash Equilibrium. The strategy and cost function of players in the game is transplanted as Algorithm (1) and utility Function (9) respectively.

3) Difference Between Equilibrium and The Aimed Variable: Here rises a question, is the final value obtained by Algorithm (1) exactly the same as the expression (15) representing a Nash equilibrium? The answer is that there is very little difference if interference is considered. Recall the target objective we want to minimize in Problem (4) is,

$$\begin{aligned}\sum_{i\in\mathcal{N}} \frac{\tilde{f}_i}{\tilde{P}_i} &= \sum_{i\in\mathcal{N}} \frac{\sum_{\substack{j\in\mathcal{N},j\neq i,\\ c(\sigma_j)=c(\sigma_i)}} \tilde{f}_{ji} + N_0}{\tilde{P}_i} \\ &= \sum_{i\in\mathcal{N}} \frac{\sum_{\substack{j\in\mathcal{N},j\neq i,\\ c(\sigma_j)=c(\sigma_i)}} \tilde{f}_{ji}}{\tilde{P}_i} + \sum_{i\in\mathcal{N}} (\frac{N_0}{\tilde{P}_i})\end{aligned} \tag{16}$$

Compare Formula (16) and (15), we find that the difference between the objective value and the final value promised by congestion game is the difference between the last items in Formula (16) and (15). When channels are evenly distributed, there is $C/N* \mid \mathcal{S} \mid \approx 1$, thus Formula (16) and (15) are approximately the same, but monotonicity on the decrease of expression (16) is not perceived whereas convergence to NE is still guaranteed. When $N_0 = 0$, the potential is exactly the same with the object we want to minimize.

From above analysis, we can see the assumption that each WBS only occupies one channel can be easily removed, for example, each WBS can access multiple channels, and we can regard that WBS consists of multiple WBSes (have the same location) and each of which works on one channel. Then the proof on convergence of WhiteCat can be applied directly to this case.

Note that the convergence of the game is independent on the the concrete form of the cost function. Using function (14), the total potential of the game is approximately the same with the total utility of all WBSes in the network, while, if the goal of a problem varies (e.g. (4) has a different objective), then a distinctive utility for each WBS need to be proposed accordingly. Hence, we say that WhiteCat scheme provides a prototype for the problems where the interaction among users are asymmetric: based on a suitable utility involves the welfare of itself and its neighborhood community, the best response approach can converge in a decentralized style.

VI. PERFORMANCE EVALUATION

We compare the performance of WhiteCat, with another two distributed heuristic schemes (Whitespace channel allocation selfish) WhiteCase and no-regret learning, besides, the centralized optimization and a random allocation are used for reference.

- *WhiteCase*: Each WBS selfishly updates its channel to achieve the best (in this paper means the smallest) possible utility based on Formula (18).
- *No-regret learning*: Each WBS maps the probability of choosing each strategy to a certain proportion of the regret which the WBS may have if it doesn't choose that strategy, and the WBS choose the strategy with the biggest probability. WBSes update such mapping dynamically and this approach converges to correlated equilibrium. Please refer the original paper [10] for details.

A 60KM x 60KM square area is divided into 16 minor square blocks evenly, for each block there is one WBS locating in the middle of it. Same mount of end terminals distributed in each minor block, however, they don't necessarily belong the WBS in that minor block, they choose the WBS to join, which caused the strongest received signal strength indicator (RSSI) on it. There is a rim area with width of 30Km around the square area, where TV contours are randomly located. The TV station which protected by TV contour working only on one channel. There are 4 TV contours belonging to 4 TV stations, each of which works on one different channel. The location of WBSes and TV contours are illustrated in Figure (4), and the other parameters are listed in the table (I).

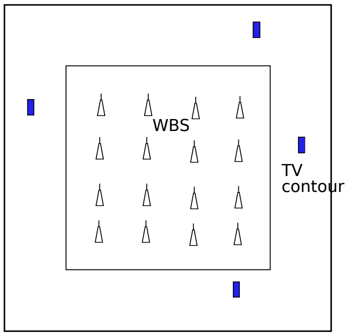

Fig. 4. Layout of WBSes and TV contours

TABLE I
SIMULATION PARAMETERS

δ	7KM
Noise	10^{-12}W
length of the square to locate WBSes	60Km
Interference threshold on TV contour	10^{-7}W
Path loss factor	2
Standard deviation in flat shadowing	8
Minimal WBS transmission power	4W
Maximal WBS transmission power	40W
Number of end terminals in network	800

A. Better Way to Decide the Maximal Transmission Power Map

We simulate the four distributed spectrum allocation schemes with the maximal permitted transmission power map obtained from convex programming and linear programming respectively, and then tell which maximal power map generation outperforms based on the performances of the four spectrum allocation schemes. We run simulations for 100 times, the WBSes' location is fixed in each run whereas the location of TV contours, end terminals and the sequence for WBS to update are randomly decided. Figure (5) elucidates that all the four distributed spectrum allocation schemes consume less transmission power consumption by from 15% (Random scheme and WhiteCat) to 35% (WhiteCase and No-regret Learning) when convex programming is applied to decide the maximal power map, meanwhile, Figure (6) shows that QuasiSINR is improved from 10% to 20% for all the four distributed spectrum allocation schemes. The cumulative distribution function curve of SINR on end terminals is drawn in Figure (7), where the x axis represents SINR level, and the y axis shows the cumulative proportion of end terminals whose SINR equals or smaller than that level. The curves show that all the four distributed schemes perform better with convex programming (the dash curves). Hence we adopt convex programming to decide the maximal transmission power in the following simulation.

B. *Comparison of Distributed Spectrum Schemes*

Now let's have a look at the performance of the four spectrum allocation approaches with convex programming to decide the maximal power map, which is elucidated in the right part of both Figure (5) and (6). We can see that WhiteCat consumes 30% less transmission power than WhiteCase and No-regret learning schemes, whereas better QuasiSINR is obtained. The cumulative distribution function curve of SINR on end terminals with convex programming is presented in Figure (7) as dash lines, we can see that for any cumulative proportion under 90%, the corresponding SINR level from Whitecat on end terminals is slightly (around 0.5-1 dB) but stably higher than that obtained by WhiteCase and No-regret schemes, and 3 dB higher than that in random scheme.

In each run of simulation, average value of the 20 % end terminals with the worst SINR is recorded, and the averaged such value over 100 simulations is illustrated in Figure (8) which shows WhiteCat achieves better performance for the worst suffered end terminals than WhiteCase and No-regret approaches.

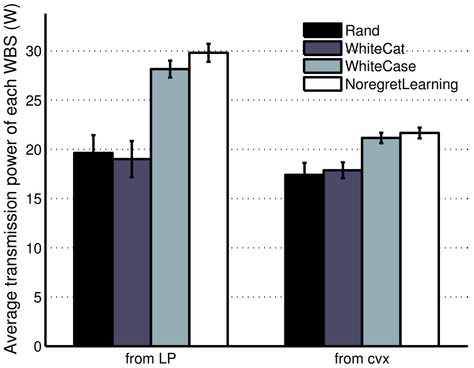

Fig. 5. Power consumed by different distributed spectrum allocation schemes under different ways deciding the maximal transmission power map

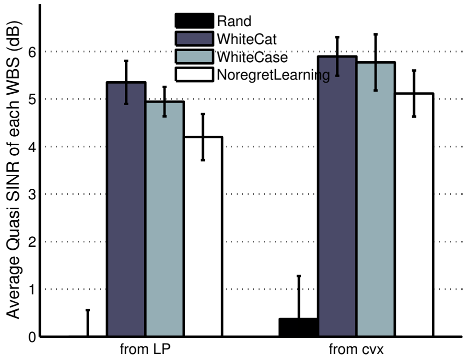

Fig. 6. QuasiSINR achieved by different distributed spectrum allocation schemes under different ways deciding the maximal transmission power map

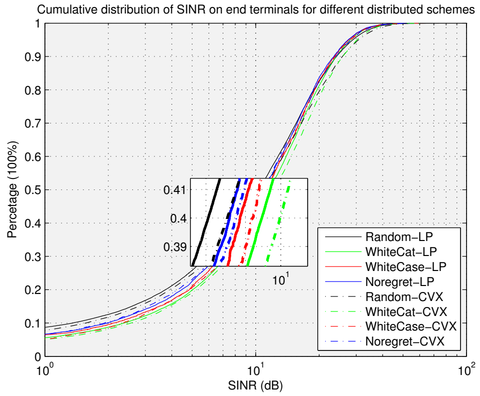

Fig. 7. CDF of SINR on end users obtained by different CA schemes under different methods to decide the maximal transmission power map

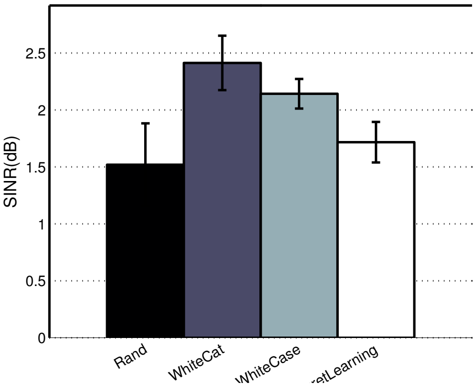

Fig. 8. The average SINR of the 20% worst end terminals

C. *Analysis on Convergence Process*

In the congestion game, each player has at most $(n-1) * |\mathcal{C}|$ resources available for usage, so there is no polynomial steps converging to NE, while, simulation shows the algorithm can quickly converge to NE when the number of WBS is up tp 100. Figure (9) depicts one instance of simulation, where WhiteCat converges quickly, No-regret produces oscillation but converges finally, while WhiteCase can not converge thus has to be stopped manually.

We also compare the convergence speed between WhteCat with no-regret scheme. We fix 16 WBSes' location working with 4 channels, whereas the location of TV contours and

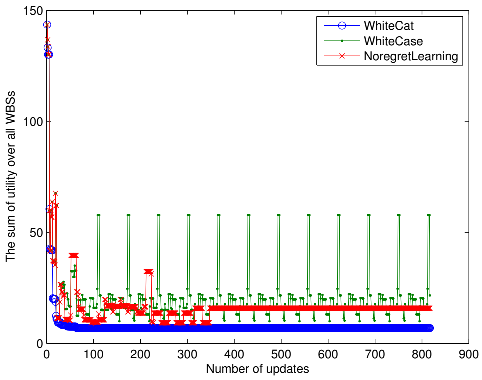

Fig. 9. Convergence with three different schemes in one simulation instance

end terminals are randomly decided. We account each WBS accessing the base station (refer to Section (V-B)) as *one step*. We record the number steps before convergence. Table (II) illustrated the average number of steps needed for convergence in 100 runs of simulations. As there is no guarantee for WhiteCase to converge, we stop the channel allocation process after 16000 steps (1000 rounds). We can see WhiteCat is 30 times faster than no-regret, and the relatively small confidence interval shows that WhiteCat's convergence is not affected obviously by different network conditions, which is reasonable as more knowledge of the network is known by users executing Whitecat. As to average running time for each convergence with Matlab, Whitecat is much smaller than the other two schemes, as the nonlinear solver LINDO to be discussed in next subsection, the running time is about 40 minutes.

TABLE II
CONVERGENCE PERFORMANCE

Scheme	average #steps	95% CI	average time
Whitecat	58	5.6	2s
Whitecase	4587	2742	50s
No-regret	1916	1541	144s
Lindo	-	-	2400s

D. Stability of SINR in the Process of Channel Allocation

WBS provides service to end users in the process of channel allocation. A certain SINR corresponds to certain transmission configurations like modulation type and data rate. Oscillation of SINR may cause reconfiguration, reduced throughput or delay variance, thus is not preferred. We propose a metric *Cost of Oscillation* (COS) to represent the stability of SINR in the converging process. We assume each update step takes the same amount of time which is 1 time unit, the variance of SINR on end user i at time point $t+1$ compared with that at time t is $\mathit{\Delta}\gamma_i(t+1) = \mid \frac{\gamma_i(t+1)-\gamma_i(t)}{\gamma_i(t)} \mid$. The COS value for one network applied with a certain channel allocation scheme is,

$$COS = \sum_{t=1}^{T} \sum_{i \in \mathcal{N}} \mathit{\Delta}\gamma_i(t) \tag{17}$$

$\gamma_i(0)$ is the SINR for i before starting channel allocation. The variance of SINR in channel allocation process is shown in table (III) from which we can see WhiteCat achieves only 6% of oscillation on SINR compared with No-regret approach.

TABLE III
STABILITY IN THE PROCESS OF CONVERGENCE

Scheme	COS	95% Confidence interval
Whitecat	8850	2984
Whitecase	246790	168050
No-regret	145460	1541

E. Comparison Between Distributed and Centralized Scheme

After comparing the performances of WhiteCat with the other two heuristic solutions, we have a look at the difference between these distributed approaches and the centralized optimization method. For these heuristic schemes, the sequence to update influences the final performance, while, it is very difficult to find out the optimal sequence which achieve the best performance, for our simulation configuration, the number of different sequence for 16 WBSes is 16! which has order of magnitude of 14. For demonstration purpose, we choose 100 different update sequences randomly for 100 times of simulation. In each simulation the sequence of WBS to update their channels is randomly decided be identical for all the 4 schemes. As solution of optimization has nothing to do with sequence, we only solve the optimization problem for once. We fixed the location of WBSes and PU contours, only leave the end terminals randomly scattered in the inner square area in each simulation.

Figure (10) shows the average power consumption and average QuasiSINR over all WBSes and rounds of simulations. WhiteCat consumes the least of power except for the random scheme. while, Lindo outperform others on QuasiSINR. Figure (11) demonstrates the cumulative distribution of SINR on all end terminals, where the centralized optimization achieves 3 dB better SINR on end terminals than distributed schemes, which means there is still big space to improve the performance of decentralized approaches.

VII. CONCLUSION

This paper proposes a solution for secondary cellular network to utilize TV white spectrum. Strictly obeying the interference restriction from primary network, the maximal permitted transmission power on each channel for each base station is decided in a centralized manner, then secondary base stations choose channel with the maximal power level on that channel distributively using WhiteCat. WhiteCat provides end terminals better SINR with less transmission power, and converges to one pure Nash equilibrium in a faster speed compared with two other schemes (greedy best response as well as a no-regret learning scheme). WhiteCat is formulated into a standard congestion game which proves the convergence of the scheme. WhiteCat requires a central data base containing information about the previously allocated channels to secondary users as well as their positions and propagation information among the

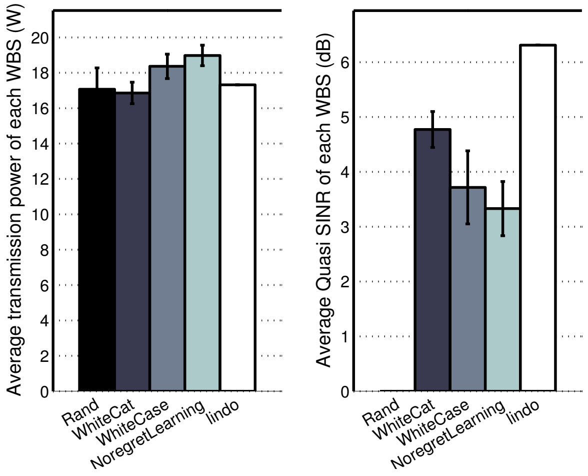

Fig. 10. Average Power consumption and QuasiSINR

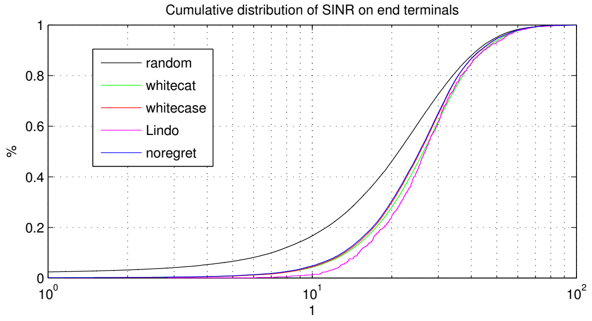

Fig. 11. Cumulative distribution of SINR on all end terminals when applying different schemes

base stations. Compared to previous work that suggests the use of a central data base for base station registration and channel validation, this is a minor overhead to be introduced. For future work we will address the problem of allowing base stations to set the transmit power arbitrarily within the maximum transmit power limit, so as to investigate further potential.

APPENDIX

PROOF OF THEOREM (1)

For selfish best response approach, the utility function is set as follows,

$$u_i = \frac{\sum_{c_i=c_i} \tilde{f_{ji}} + N_0}{P_i \cdot h_{ii}} \tag{18}$$

Proof: In order to simplify the proof, we assume $N_0 = 0$. Consider one WBS i executing algorithm (1) with utility (18), and updates its channel from c_i to c_i', we denote $u_k', k \in \mathcal{N}$ as the utility of WBS k when i chooses channel c_i', accordingly, the summation of utilities of all WBSes after i changing to c_i' is $U' = \sum_{\forall k\in\mathcal{N}, c_i=c_i'} u_k'$.

$$\begin{aligned}
U' &= u_i' + \sum_{j\in\mathcal{N}, j\neq i} u_j' \\
&= u_i' + \sum_{j\in\mathcal{N}, j\neq i} (u_j + (u_j' - u_j)) \\
&= u_i' + \sum_{j\in\mathcal{N}, j\neq i} u_j + \sum_{j\in\mathcal{N}, j\neq i} (u_j' - u_j) \\
&= u_i' + \sum_{j\in\mathcal{N}, j\neq i} u_j + \sum_{\substack{j\in\mathcal{N},\\ j\neq i,\\ c_j=c_i'}} (u_j' - u_j) + \sum_{\substack{j\in\mathcal{N},\\ j\neq i,\\ c_j=c_i}} (u_j' - u_j) \\
&\quad + \sum_{\substack{j\in\mathcal{N}, j\neq i,\\ c_j\neq c_i', c_j\neq c_i}} (u_j' - u_j) \\
&= u_i' + \sum_{j\in\mathcal{N}, j\neq i} u_j + \sum_{\substack{j\in\mathcal{N},\\ j\neq i,\\ c_j=c_i'}} (\frac{\tilde{f_{ij}}}{\tilde{P_j}}) - \sum_{\substack{j\in\mathcal{N},\\ j\neq i,\\ c_j=c_i}} (\frac{\tilde{f_{ij}}}{\tilde{P_j}})
\end{aligned} \tag{19}$$

where,

$$\begin{aligned}
u_i' &= u_i + \Delta u_i(c_i \to c_i') \\
&= u_i + \sum_{\substack{j\in\mathcal{N},\\ j\neq i,\\ c_j=c_i'}} (\frac{\tilde{f_{ji}}}{\tilde{P_i}}) - \sum_{\substack{j\in\mathcal{N},\\ j\neq i,\\ c_j=c_i}} (\frac{\tilde{f_{ji}}}{\tilde{P_i}})
\end{aligned} \tag{20}$$

bring (20) into (19), we get,

$$\begin{aligned}
U' = U &+ \sum_{\substack{j\in\mathcal{N},\\ j\neq i,\\ c_j=c_i'}} (\frac{\tilde{f_{ji}}}{\tilde{P_i}}) - \sum_{\substack{j\in\mathcal{N},\\ j\neq i,\\ c_j=c_i}} (\frac{\tilde{f_{ji}}}{\tilde{P_i}}) \\
&+ \sum_{\substack{j\in\mathcal{N},\\ j\neq i,\\ c_j=c_i'}} (\frac{\tilde{f_{ij}}}{\tilde{P_j}}) - \sum_{\substack{j\in\mathcal{N},\\ j\neq i,\\ c_j=c_i}} (\frac{\tilde{f_{ij}}}{\tilde{P_j}})
\end{aligned} \tag{21}$$

According to algorithm (1), the summation of second and third items, which is the variance of i' utility, is negative. If we can confirm the summation of fourth of the last four items is negative, the whole utility of the network decreases with i' each update. For simplification, we assume that the channel is symmetric, which means, $h_{ij} = h_{ji}$, and z is identical among all WBSes. Then, the problem we want to confirm is equivalent to the following: Given the in-equation with n, m are natural numbers

$$\sum_{i=1}^{m} \alpha_i < \sum_{i=1}^{n} \beta_i, \tag{22}$$

Prove the following in-equation is correct or not,

$$\sum_{i=1}^{m} (\alpha_i + \frac{1}{\alpha_i}) < \sum_{i=1}^{n} (\beta_i + \frac{1}{\beta_i}), \tag{23}$$

. We propose a small contradiction to prove (23) is not true. When $m = 2, n = 1$, and $\alpha_1 = 1, \alpha_2 = 0.5, \beta = 2.1$, we can see that although $\sum_{i=1}^{m} \alpha_i = 1.5 < \sum_{i=1}^{n} \beta_i = 2.1$, there is $\sum_{i=1}^{m} (\alpha_i + \frac{1}{\alpha_i}) = 4.5 > \sum_{i=1}^{n} (\beta_i + \frac{1}{\beta_i}) = 2.58$. hence, with

WBS's update, it is possible that $U' > U$, thus there is no monotonically convergence by utilizing (18). ■

Notice that the last four items in (21) is exactly the change of summation of utilities of all WBSes after i' update if WhiteCat is executed, hence the monotonic convergence of WhiteCat is proved here analytically if noise is considered to be zero. If noise is considered, we can follow the conclusion in the end of (V-C2) that WhiteCat converges without monotonicity.

DEVIATION OF PROBLEM (7)

we reformulate the objective Problem (7) here,

$$
\begin{aligned}
&\sum_{i=1}^{N} \frac{\sum\limits_{j\in\mathcal{N}, j\neq i} P^T X_j (X_j^T X_i) h_{jQ} z_{jQ} + N_0}{P^T X_i h_{iQ}} \\
&= \sum_{i=1}^{N} \Big(\frac{\sum\limits_{j\in\mathcal{N}, j\neq i} \sum\limits_{k} (P_{jk} \cdot x_{jk} \cdot x_{ik} \cdot x_{jk} \cdot h_{jQ} \cdot z_{jQ}) + \sum\limits_{k} N_0 \cdot x_{ik}}{P^T X_i h_{iQ}} \Big) \\
&= \sum_{i=1}^{N} \Big(\frac{\sum\limits_{j\in\mathcal{N}, j\neq i} \sum\limits_{k} (P_{jk} \cdot x_{jk} \cdot x_{ik} \cdot h_{jQ} \cdot z_{jQ})}{P^T X_i h_{iQ}} + \frac{\sum\limits_{k} N_0 \cdot x_{ik}}{P^T X_i h_{iQ}} \Big) \\
&= \sum_{i=1}^{N} \Big(\sum_{j\in\mathcal{N}, j\neq i} \sum_{k} \frac{(P_{jk} \cdot x_{jk} \cdot x_{ik} \cdot h_{jQ} \cdot z_{jQ})}{P^T X_i h_{iQ}} + \sum_{k} \frac{N_0 \cdot x_{ik}}{P^T X_i h_{iQ}} \Big)
\end{aligned} \tag{24}
$$

we now simplify the first item in the parenthesis. If we assume secondary base station i is working on channel m, then there is $x_{im} = 1$, and wen get,

$$
\begin{aligned}
\frac{P_{jk} \cdot x_{jk} \cdot x_{ik} \cdot h_{jQ} \cdot z_{jQ}}{P^T X_i h_{iQ}} &= \frac{P_{jk} \cdot x_{jk} \cdot x_{im} \cdot h_{ji} \cdot z}{P_{im} \cdot x_{im} \cdot h_{iQ}} \\
= \frac{P_{jk} \cdot x_{jk} \cdot h_{jQ} \cdot z_{jQ}}{P_{im} \cdot h_{iQ}} &
\end{aligned} \tag{25}
$$

other wise, Formula (25) equals to 0.

Similarly, for the second item in the bracket,

$$
\frac{N_0 \cdot x_{ik}}{P^T X_i h_{iQ}} = \frac{N_0}{P_{ik} h_{iQ}} \cdot x_{ik} \tag{26}
$$

then, Formula (24) becomes,

$$
\sum_{i=1}^{n} \Big(\sum_{j\in\mathcal{N}, j\neq i} \sum_{k} \frac{P_{jk}}{P_{ik} h_{iQ}} \cdot h_{jQ} \cdot z_{jQ} \cdot x_{jk} \cdot x_{ik} + \sum_{k} \frac{N_0}{P_{ik} h_{iQ}} \cdot x_{ik} \Big), \tag{27}
$$

this is a binary quadratic programming problem.

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