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Effective Service Capacity Analysis of Interference-Limited Multi-Carrier Wireless Systems

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***Abstract*—A good utilization of a wireless network while still ensuring QoS-constraints for all users is a hard challenge for service providers. Especially for admission control or handover decisions it is important to have a good approximation of the possible rate a user can achieve with respect to QoS parameters like delay or outage probabilities. Since fourth generation cellular networks utilize a frequency reuse of one, especially terminals at the cell edge suffer from inter-cell interference. In order to provide them with rates that guarantee specific QoS parameters without reserving too much resources, a good prediction of their possible rates is needed. Unfortunately, the prediction of the rates in an interference-limited cell is a complex and hard problem. Hence, it is a common way to simplify calculations by treating interference as additional noise in the used system models. In this paper we derive closed-form solutions for the delay distribution of interference-limited cells with respect to an OFDMA and a round robin scheduling approach. These distributions can be used to predict the possible rates of users given their average received transmitter gains, interferer gains and their QoS-constraints in a very accurate way. We validate our derivations and show that a simplification by treating interference as noise leads to an underestimation of rates which lowers the cell throughput. Furthermore, we show that due to the resulting equations, there is no way to derive the rates for interference limited cells in a linear way from easier solutions given by noise-limited cells.**

I. INTRODUCTION

Upcoming cellular networks of the fourth generation are going to exhibit two extraordinary circumstances which have not been witnessed in cellular networks before. On the one hand, these systems are designed to be interference-limited as neighboring cells of the same network provider are going to operate in the same frequency band. Although there are built-in methods to reduce the amount of interference, these cells are exposed to (by coordinating either the transmit powers per resource block [1] or the applied beamforming patterns [5]), there is nevertheless a significant amount of interference that neighboring cells will experience. Interference-limited systems are not easy to model and analyse, as the stochastics of the signal-of-interest and of the interfering signals need to be carefully modelled jointly with respect to the link-to-system function. Due to this involved analysis, it is common to see the impact of interference being modelled as a noise-equivalent term (i.e. accounting for the interference as if it was an additional noise source and adjusting the corresponding power by the average interference power).

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In addition, future cellular networks need to meet the demands of a fast increasing real-time related traffic. This is already evident as video-conferencing and streaming applications are used by cellular customers today. The usage of these applications is expected to increase significantly over the next years. However, with the advent of machine-to-machine communication applications, there might be even more demanding traffic flows present in the systems for which tough quality-of-service demands need to be met. In the context of both of these circumstances of fourth generation cellular networks, the question arises which service capabilities cellular networks offer and how to model and derive (predict) them in order to perform for example tasks like admission control, radio resource management or flow control. While fundamental models are required that can predict the queuing performance of such systems, the interference-limitation together with dynamic resource allocation at the base station make the analysis of the systems quite tough.

In this paper, we address these fundamental issues. For our analysis we rely on the approximation framework of the *effective service capacity* introduced by Wu et. al. in [10] which was later on extended by Soret et. al. in [8]-[9]. The effective service capacity allows to analytically obtain the queuing delay distribution by carefully analysing the service process of a communication system. Especially the simplification in the analysis introduced by Soret [8] allows the consideration of quite complex systems. Hence, we are able to obtain approximations of the delay for interference-limited multi-carrier wireless systems under two different scheduling policies in this paper: under opportunistic OFDMA scheduling (as a representative of a dynamic scheduler) as well as round robin scheduling (as a representative of a static approach). To the best of our knowledge, these results are novel with respect to state-of-the-art. After deriving the results, we first validate them and afterwards perform a numerical evaluation of the obtained analytical results. We can show that on the one hand the queuing performance of the investigated interference-limited systems can be accurately predicted by our results. On the other hand, the results demonstrate that modelling interference power as an additional noise source is a huge modelling mistake, underestimating the performance of interference-limited systems quite strongly.

The rest of the paper is structured in the following way. In Section II, we first introduce our system model, state the problem formulation and give a brief summary of the effective service capacity framework. In Section III, we then present our main contribution which is the analysis of the two scheduling policies for the interference-limited case. In Section IV, we first validate our analysis before we show numerical results of the system behaviour in Section V. We finally present some conclusions in Section VI.

II. PRELIMINARIES

In this section we first introduce the system model, followed by the formulation of the problem we are dealing with. Finally, we give a brief review of the effective service capacity framework.

A. System Model

We consider the down-link of a cell in a larger cellular network deployment. The considered cell contains J different terminals. All data transmissions (up-link as well as down-link) are coordinated by the base station (BS). The system features a multi-carrier transmission scheme with a total bandwidth of B [Hz] which is divided into N resource blocks (RBs). Time is slotted into frames with a frame-duration of T_{f} [s]. During one frame the base station transmits $\mathcal{S}$ symbols, each with a symbol duration of $T_{\mathcal{S}}$ [s]. For data transmission, the BS can apply a maximum transmit power of $P_{\mathrm{TX}}^{\mathrm{S}}$ [W] per RB. Interference is modelled by a neighboring base station that emits a maximum transmit power of $P_{\mathrm{TX}}^{\mathrm{I}}$ [W].

A constant data stream with rate r_j [bits/frame] arrives for every terminal $j \in J$ and is buffered at the BS for transmission. Each stream has certain quality-of-service requirements given by the tuple $\{d_j, \mathcal{P}_j\}$ with d_j denoting the maximum tolerable delay of each bit of the data flow j. This maximum delay is allowed to be violated with a maximum probability of $\mathcal{P}_j$. We refer to this as the outage probability for flow j. This service model matches very well interactive multimedia applications like voice or video telephony where the majority of the data packets (i.e. a share of $1 - \mathcal{P}_j$) needs to reach the destination within a predefined deadline d_j. Prior to each frame the terminals transmit their current channel states on all RBs to the BS. The channel state is a random variable and it is characterized by the instantaneous SINR $\gamma_{j,n} = P_{\mathrm{TX}}^{\mathrm{S}} \cdot g_{j,n}^{\mathrm{S}} / (P_{\mathrm{TX}}^{\mathrm{I}} \cdot g_{j,n}^{\mathrm{I}} + \sigma^2)$. Here, $g_{j,n}^{\mathrm{S}}$ denotes the instantaneous channel gain of the signal of interest (from the serving base station) for RB n to terminal j while $g_{j,n}^{\mathrm{I}}$ denotes the instantaneous channel gain of the interfering signal. Both channel gains are modelled as exponentially distributed random variables and are assumed to be statistically independent. In addition, the individual channel gains are also spatially independent as well as independent in time and frequency. The noise power per RB is assumed to be constant and denoted by σ^2. We assume that due to slow varying power control at the BS the average values $\mathrm{P}_{\mathrm{S}} := P_{\mathrm{TX}}^{\mathrm{S}} \cdot g_{j,n}^{\mathrm{S}}$ of the signal of interest are kept equal for all N RB and all J terminals (power control can react to the variation of shadowing, but not to fast fading).

There are several ways a scheduler can be implemented in order to assign the RBs to the terminals. We distinguish two schemes in this paper, a static and a dynamic scheme. In the static approach the RBs are assigned to the terminals regardless of the instantaneous channel states. While this reduces the overhead of transmitting the channel assignments previous to a data transmission, a static scheme is not able to exploit the positive effects of multiuser diversity. The dynamic approach is contrary to the static one. Here, the RBs are assigned to the terminal which has the best channel conditions. In that way the RB is optimally used, for the price of transmitting the actual RB assignments previous to each transmission slot.

As static scheme we use a very simple one. The RBs are assigned in a *round robin* fashion, i.e. in one frame all RBs are assigned exclusively to one terminal, and the terminals are treated in a periodic fashion. One service period lasts until every terminal was in charge to transmit its data. Therefore, the service period of the round robin approach lasts $J \cdot T_{\mathrm{f}}$. In the second set-up, the dynamic one, the RBs are assigned by an *opportunistic OFDMA* (orthogonal frequency division multiple access) scheduler, i.e. per frame for each RB the terminal that has the best instantaneous SINR is assigned. Hence, per frame at most N terminals can be served in parallel as the RB is the smallest assignable unit in frequency.

In both set-ups the RBs are modulated in a *discrete adaptive way*. Adaptive means that each RB is individually modulated according to its instantaneous SINR. Therefore, the SINR values are divided into M different ranges, and each range $[\gamma_i, \gamma_{i+1})$, $i \in \{0, \ldots, M-1\}$, ($\gamma_0 = 0$, $\gamma_M = \infty$) is modulated with a specific MCS which allows the transmission of c_i bits per symbol. Due to the feedback from the terminals, for each RB and each terminal the scheduler knows the instantaneous SINR before allocating the resources. For the mapping from the SINR γ_i to the transport capacity c_i we consider the one proposed in [3] which matches the performance of LTE systems.

B. Problem Statement

In this paper we are interested in characterizing the effective service capacity of interference-limited multi-carrier systems under static and opportunistic scheduling. The effective service capacity is an analytical approximation of the queuing performance of a system. Amongst others, it allows to derive the maximum rate a flow can have such that the QoS-pair of outage probability and delay can still be supported. The advantage of the approximation is the analytical relationship between this maximum arrival rate and all performance-relevant parameters of the system (like transmit power, scheduling algorithm, behaviour of the channel gains etc.). Hence, obtaining the capacity allows e.g. performing admission control or could serve for other system tasks like handover decisions and radio resource management. The difficulty of the addressed problem lies in the derivation, as especially interference-limited systems are not easy to model and analyse. A further complication comes from the consideration of a dynamic resource scheduler at the base station (opportunistic scheduling) which has a significant impact on the distribution of the SINR of the *scheduled* RBs.

C. Effective Service Capacity Framework

In the following, we give a very brief overview of the used framework of the effective service capacity. For additional information and a more detailed overview we refer to [8]-[10].

The effective service capacity framework is used to characterize the queue-length for constant arrival processes and arbitrary service processes. In the following, $s_j[t]$ denotes the

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service rate for terminal j at time t, and $S_j[i] = \sum_{t=1}^{i} s_j[t]$ represents the corresponding cumulative service process. We assume that the service process, as well as the increments of the cumulative service process $S_j[i]$, are stationary.

Based on the *effective bandwidth theory*, Wu et. al. derived in [10] a formula for the relationship of the arrival rate r_j and a so-called QoS-exponent θ:

$$r_j < -\frac{\Lambda(-\theta)}{\theta}, \tag{1}$$

where $\Lambda(\theta)$ denotes the log-moment generating function of the cumulative service process $S_j[i]$, defined as

$$\Lambda(\theta) = \lim_{i\to\infty} \frac{1}{i} \log \mathrm{E}\left[e^{\theta\cdot(S_j[i]-S_j[1])}\right]. \tag{2}$$

The effective service capacity refers to the ratio $-\Lambda(-\theta)/\theta$, as a stable finite queue length can only be guaranteed if the constant arrival rate r_j of the source is smaller than the service capacity $-\Lambda(-\theta)/\theta$.

Utilizing the framework requires the derivation of the effective service capacity of the service process, which can be a very tough task. However, Soret et. al. extended the theory in [8]-[9] to more common expressions of the service process, namely mean and variance. Based on the Chernoff Bound, they get the following connection of the outage probability $\mathcal{P}_j$, delay of the current head-of-line bit D_j, maximum delay d_j and QoS-exponent θ:

$$\mathcal{P}_j = \mathrm{Pr.}\left\{D_j > d_j\right\} \approx K \cdot e^{-\theta\cdot r_j\cdot d_j}, \tag{3}$$

where $K \in [0,1]$ denotes the probability that the queue is not empty. If the increments of the cumulative service process $S_j[i]$ are i.i.d., Equation (1) can be restated according to [7]:

$$r_j < \mathrm{E}\left[s_j[i]\right] - \frac{\theta}{2}\mathrm{Var}\left[s_j[i]\right]. \tag{4}$$

This equation also includes the QoS-exponent, but in combination with Equation (3), a given d_j and a given $\mathcal{P}_j$ it can be used to derive a maximum sustainable rate, i.e. an upper bound for an arrival rate that can approximatively meet the QoS-constraints with

$$r_j^* \approx \frac{1}{2} \cdot \left(\mathrm{E}\left[s_j\right] + \sqrt{(\mathrm{E}[s_j])^2 + \frac{2\cdot \ln(\mathcal{P}_j)}{d_j}\cdot \mathrm{Var}[s_j]}\right). \tag{5}$$

Since the variance can be derived by the first and second moment with $\mathrm{Var}[s_j] = \mathrm{E}[s_j^2] - (\mathrm{E}[s_j])^2$, it suffices to estimate these first two moments. As shown in [9], these formulas can also be used for correlated channels. Here the increments of the cumulative service process are rearranged to make them i.i.d.. While this implies no big effort for the computation of the mean, the derivation of the corresponding variance becomes complicated. However, in the following we will derive the moments for the previously introduced opportunistic OFDMA and round robin schemes with uncorrelated channels, leaving the correlated case as future work.

III. Effective Capacity Derivations

In this section, we derive the first and second moment of the instantaneous service process increments s_j. These derivations are done for the round robin and the opportunistic scheduler. Due to the assumption that the system utilizes discrete modulation and coding schemes, the derivation of the interference-limited moments of the service process becomes much easier to calculate. It simply consists of adding up the solutions of definite integrals multiplied with the constant capacity c_i for this region. If we consider M different modulation/coding combinations, the expected value equals

$$\mathrm{E}[s_j] = \sum_{i=0}^{M-1} c_i \cdot \int_{\gamma_i}^{\gamma_{i+1}} p(\gamma)d\gamma, \tag{6}$$

with $\gamma_0 = 0$, $\gamma_M = \infty$, and $p(\gamma)$ denoting the PDF of the SINR.

A. Round Robin with Adaptive Modulation

We start with the analysis of the round robin system. Recall that in this case the queues of the terminals are served one by one by assigning all RBs during one frame exclusively to one terminal. In order to derive the mean and the second moment of the service increments, we first need the distribution function of the SINR of a RB. As published in [4], the CDF and PDF of the SINR $\gamma_{j,n}$ for a RB n for terminal j are given as follows (assuming that only one interferer is present):

$$f_{\gamma_{j,n}}(x) = \left[\frac{\sigma^2}{\mathrm{P_I}x + \mathrm{P_S}} + \frac{\mathrm{P_I P_S}}{(\mathrm{P_I}x + \mathrm{P_S})^2}\right] \cdot e^{-\frac{\sigma^2}{\mathrm{P_S}}x} \tag{7}$$

$$F_{\gamma_{j,n}}(x) = 1 - \frac{\mathrm{P_S}}{\mathrm{P_I}x + \mathrm{P_S}} \cdot e^{-\frac{\sigma^2}{\mathrm{P_S}}x}, \tag{8}$$

with $\mathrm{P_I}$ denoting the average over $P_{\mathrm{TX}}^{\mathrm{I}} \cdot g_{j,n}^{\mathrm{I}}$. Since all RBs are assigned to one terminal per frame, the expected capacity of the random service s_j for terminal j is obtained as:

$$\begin{aligned}\mathrm{E}[s_j] &= \mathcal{S} \cdot \sum_{n=1}^{N} \sum_{i=0}^{M-1} c_i \int_{\gamma_i}^{\gamma_{i+1}} f_{\gamma_{j,n}}(x)dx \\ &= N \cdot \mathcal{S} \cdot \sum_{i=0}^{M-1} c_i \cdot \left[1 - \frac{\mathrm{P_S}}{\mathrm{P_I}x + \mathrm{P_S}} \cdot e^{-\frac{\sigma^2 x}{\mathrm{P_S}}}\right]_{\gamma_i}^{\gamma_{i+1}}.\end{aligned} \tag{9}$$

For the second moment, we get the following solution:

$$\begin{aligned}\mathrm{E}\left[s_j^2\right] &= N \cdot \mathrm{E}\left[s_{j,n}^2\right] + (N^2 - N) \cdot (\mathrm{E}\left[s_{j,n}\right])^2 \\ &= N \cdot \mathcal{S}^2 \cdot \sum_{i=0}^{M-1} c_i^2 \int_{\gamma_i}^{\gamma_{i+1}} f_{\gamma_{j,n}}(x)dx \\ &\quad + (N^2 - N) \cdot (\mathrm{E}\left[s_{j,n}\right])^2 \\ &= N \cdot \mathcal{S}^2 \cdot \sum_{i=0}^{M-1} c_i^2 \cdot \left[1 - \frac{\mathrm{P_S}}{\mathrm{P_I}x + \mathrm{P_S}} \cdot e^{-\frac{\sigma^2 x}{\mathrm{P_S}}}\right]_{\gamma_i}^{\gamma_{i+1}} \\ &\quad + (N^2 - N) \\ &\quad \cdot \left(\mathcal{S} \cdot \sum_{i=0}^{M-1} c_i \cdot \left[1 - \frac{\mathrm{P_S}}{\mathrm{P_I}x + \mathrm{P_S}} \cdot e^{-\frac{\sigma^2 x}{\mathrm{P_S}}}\right]_{\gamma_i}^{\gamma_{i+1}}\right)^2.\end{aligned} \tag{10}$$

These moments hold for the total service period which lasts for J frames.

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B. OFDMA with Adaptive Modulation

For the opportunistic OFDMA scheduler the derivation of the distribution function is more challenging. This is clearly due to the dynamic resource allocation which modifies the distribution of the SINRs in a favourable way. Since under opportunistic scheduling the terminal with the best instantaneous SINR gets the RB assigned, the distribution function can be obtained based on order statistics [6], which describe the CDF and the PDF of the best RB out of J independent but not identically distributed ones as

$$\begin{aligned} F^*_{(J/J)}(x) &= \prod_{j=1}^{J} F_{\gamma_{j,n}}(x) \\ &= \prod_{j=1}^{J} \left(1 - \frac{\mathrm{P_S}}{\mathrm{P}_{\mathrm{I},j}x + \mathrm{P_S}} \cdot e^{-\frac{\sigma^2 x}{\mathrm{P_S}}}\right) \end{aligned} \tag{11}$$

and

$$f^*_{(J/J)}(x) = \sum_{j=1}^{J} F'_{\gamma_{j,n}}(x) \prod_{\substack{i=1 \\ j\neq i}}^{J} F_{\gamma_{i,n}}, \tag{12}$$

with $\mathrm{P}_{\mathrm{I},j}$ denoting the average over $P^{\mathrm{I}}_{\mathrm{TX}} \cdot g^{\mathrm{I}}_{j,n}$ of terminal j. In the following, we consider the special case that the average received interfering power is also equal for all terminals. However, the moments for the general case with independent but not identically distributed random variables can be achieved by replacing $f_{(J/J)}(x)$ with $f^*_{(J/J)}(x)$ in Equation (15) and in Equation (17). For identically distributed interferer gains the CDF and PDF evolve to

$$\begin{aligned} F_{(J/J)}(x) &= \left(F_{\gamma_{j,n}}(x)\right)^J \\ &= \left(1 - \frac{\mathrm{P_S}}{\mathrm{P_I}x + \mathrm{P_S}} \cdot e^{-\frac{\sigma^2 x}{\mathrm{P_S}}}\right)^J \end{aligned} \tag{13}$$

and

$$\begin{aligned} f_{(J/J)}(x) &= J \cdot F^{J-1}_{\gamma_{j,n}}(x) \cdot f_{\gamma_{j,n}}(x) \\ &= J \cdot \left(1 - \frac{\mathrm{P_S} \cdot e^{-\frac{\sigma^2 x}{\mathrm{P_S}}}}{\mathrm{P_I}x + \mathrm{P_S}}\right)^{J-1} \\ &\quad \cdot \left[\frac{\sigma^2}{\mathrm{P_I}x + \mathrm{P_S}} + \frac{\mathrm{P_I P_S}}{(\mathrm{P_I}x + \mathrm{P_S})^2}\right] \cdot e^{-\frac{\sigma^2 x}{\mathrm{P_S}}} \\ &= J \cdot \sum_{i=0}^{J-1} \binom{J-1}{i} (-1)^i \\ &\quad \cdot \left[\frac{\sigma^2 \mathrm{P}_\mathrm{S}^i}{(\mathrm{P_I}x + \mathrm{P_S})^{i+1}} + \frac{\mathrm{P_I P}_\mathrm{S}^{i+1}}{(\mathrm{P_I}x + \mathrm{P_S})^{i+2}}\right] \\ &\quad \cdot e^{-\frac{\sigma^2 (i+1) x}{\mathrm{P_S}}}, \end{aligned} \tag{14}$$

respectively. Using these distributions we can derive the first and second moment for the opportunistic OFDMA set-up in the following way:

$$\begin{aligned} \mathrm{E}[s_j] &= \sum_{k=1}^{N} \mathcal{P}(X_j = k) \cdot k \cdot \mathrm{E}\left[s_j^{(J/J)}\right] \\ &= \frac{N}{J} \cdot \mathcal{S} \cdot \sum_{i=0}^{M-1} c_i \int_{\gamma_i}^{\gamma_{i+1}} f_{(J/J)}(x) dx \qquad (15) \\ &= \frac{N}{J} \cdot \mathcal{S} \cdot \sum_{i=0}^{M-1} c_i \int_{\gamma_i}^{\gamma_{i+1}} J \cdot \left(1 - \frac{\mathrm{P_S} \cdot e^{-\frac{\sigma^2 x}{\mathrm{P_S}}}}{\mathrm{P_I}x + \mathrm{P_S}}\right)^{J-1} \\ &\quad \cdot \left[\frac{\sigma^2}{\mathrm{P_I}x + \mathrm{P_S}} + \frac{\mathrm{P_I P_S}}{(\mathrm{P_I}x + \mathrm{P_S})^2}\right] \cdot e^{-\frac{\sigma^2 x}{\mathrm{P_S}}} dx \\ &= \frac{N}{J} \cdot \mathcal{S} \cdot \sum_{i=0}^{M-1} c_i \cdot \int_{\gamma_i}^{\gamma_{i+1}} J \cdot \sum_{l=0}^{J-1} \binom{J-1}{l} (-1)^l \\ &\quad \cdot \left[\frac{\sigma^2 \mathrm{P}_\mathrm{S}^l}{(\mathrm{P_I}x + \mathrm{P_S})^{l+1}} + \frac{\mathrm{P_I P}_\mathrm{S}^{l+1}}{(\mathrm{P_I}x + \mathrm{P_S})^{l+2}}\right] \cdot e^{-\frac{\sigma^2 (l+1) x}{\mathrm{P_S}}} dx \\ &= N \cdot \mathcal{S} \cdot \sum_{i=0}^{M-1} c_i \cdot \sum_{l=0}^{J-1} \binom{J-1}{l} (-1)^l \\ &\quad \cdot \int_{\gamma_i}^{\gamma_{i+1}} \left[\frac{\sigma^2 \mathrm{P}_\mathrm{S}^l}{(\mathrm{P_I}x + \mathrm{P_S})^{l+1}} + \frac{\mathrm{P_I P}_\mathrm{S}^{l+1}}{(\mathrm{P_I}x + \mathrm{P_S})^{l+2}}\right] \\ &\quad \cdot e^{-\frac{\sigma^2 (l+1) x}{\mathrm{P_S}}} dx \\ &= N \cdot \mathcal{S} \sum_{i=0}^{M-1} c_i \cdot \sum_{l=0}^{J-1} \binom{J-1}{l} (-1)^l \cdot \frac{\mathrm{P}_\mathrm{S}^l}{l+1} \\ &\quad \cdot \left[\frac{e^{-\frac{\sigma^2 (l+1)\gamma_i}{\mathrm{P_S}}}}{(\mathrm{P_I}\gamma_i + \mathrm{P_S})^{l+1}} - \frac{e^{-\frac{\sigma^2 (l+1)\gamma_{i+1}}{\mathrm{P_S}}}}{(\mathrm{P_I}\gamma_{i+1} + \mathrm{P_S})^{l+1}}\right] \qquad (16) \end{aligned}$$

and

$$\begin{aligned} \mathrm{E}\left[s_j^2\right] &= \frac{N}{J} \cdot \mathcal{S}^2 \cdot \sum_{i=0}^{M-1} c_i^2 \int_{\gamma_i}^{\gamma_{i+1}} f_{(J/J)}(x)\, dx \\ &\quad + \left(1 - \frac{1}{N}\right) (\mathrm{E}[s_j])^2 \qquad (17) \\ &= N \cdot \mathcal{S}^2 \cdot \sum_{i=0}^{M-1} c_i^2 \cdot \sum_{l=0}^{J-1} \binom{J-1}{l} (-1)^l \cdot \frac{\mathrm{P}_\mathrm{S}^{l+1}}{l+1} \\ &\quad \cdot \left[\frac{e^{-\frac{\sigma^2 (l+1)\gamma_i}{\mathrm{P_S}}}}{(\mathrm{P_I}\gamma_i + \mathrm{P_S})^{l+1}} - \frac{e^{-\frac{\sigma^2 (l+1)\gamma_{i+1}}{\mathrm{P_S}}}}{(\mathrm{P_I}\gamma_{i+1} + \mathrm{P_S})^{l+1}}\right] \\ &\quad + \left(1 - \frac{1}{N}\right) (\mathrm{E}[s_j])^2. \qquad (18) \end{aligned}$$

IV. Validation

In this section we validate our derivations by simulations. All simulations are done in Matlab. First, we validate the formulas for the mean and the second moment, followed by a validation of the resulting maximum sustainable rates for the interference-limited case. Note that we follow here the

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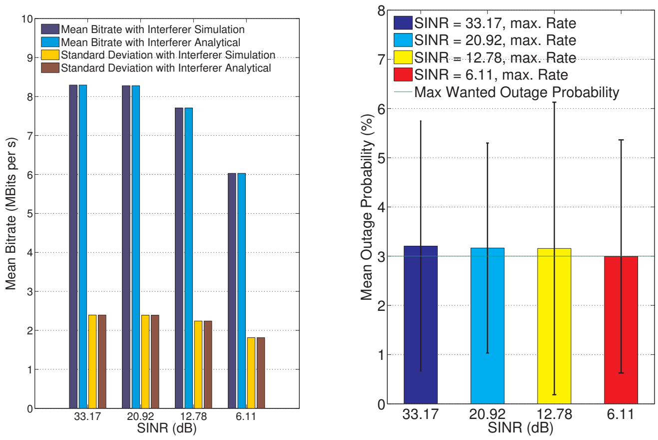
approach in [2], where we have presented results for noise-limited scenarios.

We used the following scenario parameters for the validation. There are $J = 5$ terminals per cell which are served by a total number of $N = 48$ RBs. A frame lasts $T_\mathrm{f} = 1$ ms, and per frame $\mathcal{S} = 144$ symbols can be transmitted per RB. We simulated $800,000$ sequential frames, which correspond to about 13 minutes of real time, and each scenario simulation was repeated 30 times. A total power of 40 Watt is equally distributed to the RBs with a noise power of -112 dBm. As received power for the signal of interest and the interfering signal, respectively, we used the following 4 tuples: $\{-66\,\mathrm{dBm}, -99.4\,\mathrm{dBm}\}$, $\{-76.5\,\mathrm{dBm}, -97.6\,\mathrm{dBm}\}$, $\{-82.7\,\mathrm{dBm}, -95.5\,\mathrm{dBm}\}$ and $\{-87\,\mathrm{dBm}, -93.2\,\mathrm{dBm}\}$. This results in average SINR values at the terminals of 33.17, 20.92, 12.78 and 6.11 dB.

Figure 1 holds the results for the opportunistic OFDMA case. As shown in Figure 1a, there is a perfect match of the analytical means and second moments and the simulation results. Figure 1b presents the resulting outage probabilities that arise if we simulate the usage of the calculated maximum sustainable rate for a maximum delay of 50 ms and a required outage probability of 3 % as a constant arrival rate. The calculated rates per terminal are displayed in Table I. As shown in [2], the analytically derived maximum sustainable rate is rather an upper bound for the possible rate, hence, it is not surprising that the resulting outage probabilities are slightly above the target one. Note that no confidence intervals are displayed but the highest and lowest observed outage probabilities. This is necessary since we want to guarantee



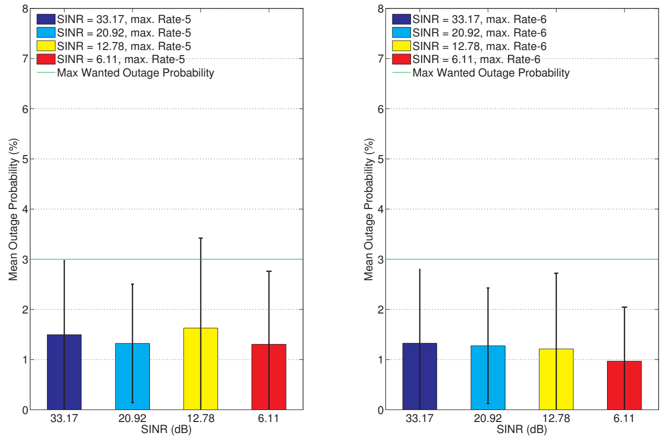
(a) Mean and second moment (b) Outage probabilities for r^*

Fig. 1. Results for OFDMA: (a) shows a comparison between simulated and calculated mean and second moment, (b) presents the outage probabilities for the calculated maximum sustainable rate assuming a maximum delay of 50 ms and a target outage probability of 3 %.

that the outage probabilities are fulfilled.

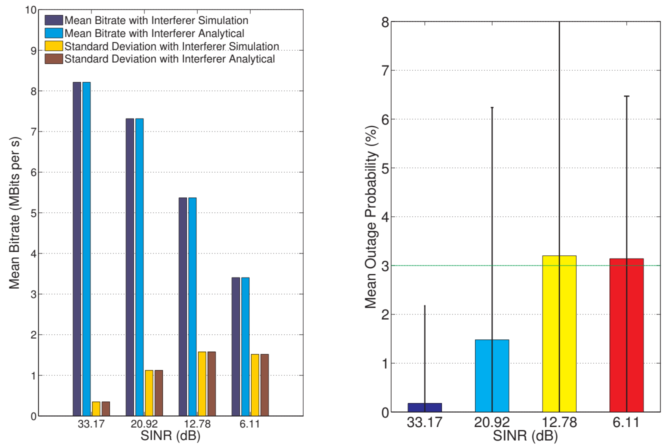
TABLE I. CALCULATED MAXIMUM SUSTAINABLE RATES PER TERMINAL IN MBIT/S FOR 5 TERMINALS, 48 RBS, 144 SYMBOLS PER FRAME, 50 MS DELAY AND AN OUTAGE PROBABILITY OF 3 % FOR THE FOUR USED SINR VALUES.

SINR (dB)	33.17	20.92	12.78	6.11
OFDMA: r^*	8.27	8.253	7.681	6.011
RR(per s): r^*	8.21	7.32	5.37	3.40



(a) Outage Probabilities for r^*-5 (b) Outage Probabilities for r^*-6

Fig. 2. This figure shows the outage probabilities simulated for the OFDMA-setup used before, but now with a reduced arrival rate. Figure 2a shows the results for the maximum sustainable rate lowered by 5 bits/ms, Figure 2b displays the outcomes for a rate lowered by 6 bits/ms.



(a) Mean and Second Moment (b) Outage Probabilities

Fig. 3. Results for round robin approach: (a) shows a comparison between simulated and calculated mean and standard deviation , (b) presents the outage probabilities for the calculated maximum sustainable rate assuming a maximum delay of 50 ms and a wanted outage probability of 3 %.

In order to find the rate that does not longer violate our outage probability goal, we repeat the simulations with reduced rates. As shown in Figure 2, a reduction of 5 bits per millisecond is still slightly above the target outages, whereas a reduction of 6 bits/ms totally fulfilled the outage condition. Note that a reduction of 6 bits is less than 0.1 % of the calculated rates. Hence, the maximum sustainable rate is still a very good approximation of the rate the system could provide.

Next, we consider the round robin system. Also in the round robin case, there is a perfect match of the analytical first and second moment (used for standard deviation calculation) and the corresponding results from simulations (see Figure 3a). Astonishing is the slight difference between the mean values for round robin and OFDMA for an average SINR of 33.17 dB. This is caused by the fact that the spectral efficiency of the discrete modulation/coding schemes saturates above 30 dB at around 6 bit/symbol. Hence, there is no gain from dynamic resource allocation to achieve. Also the results for the maxi-

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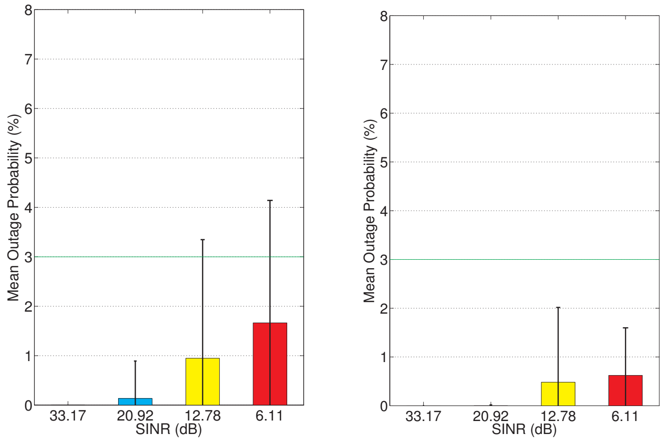


(a) Outage Probabilities for r^*-1 (b) Outage Probabilities for r^*-2

Fig. 4. Results for round robin cases with 5 terminals, 50 ms delay and $\mathcal{P}_j = 3\%$. As arrival rate we use the maximum sustainable rate lowered by 1 bit/ms and 2 bit/ms respectively.

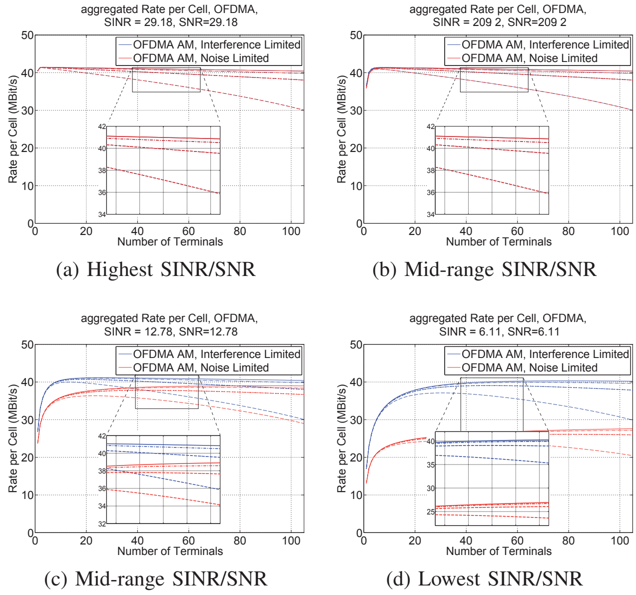
mum sustainable rates are similar to the OFDMA results, but for round robin only the rates calculated for a lower SINR tend to violate the outage probabilities (compare Figure 3b for round robin and Figure 1b for OFDMA). For the round robin schemes, lowering the maximum sustainable rate by 2 bits only suffices to fulfil the target outage probability.

V. Numerical Evaluation

After the validation of our derived equations, we will now have a look at the consequences considering the aggregated maximum sustainable rates in interference- and noise-limited cells.

A. Differences between Noise- and Interference-Limited Systems

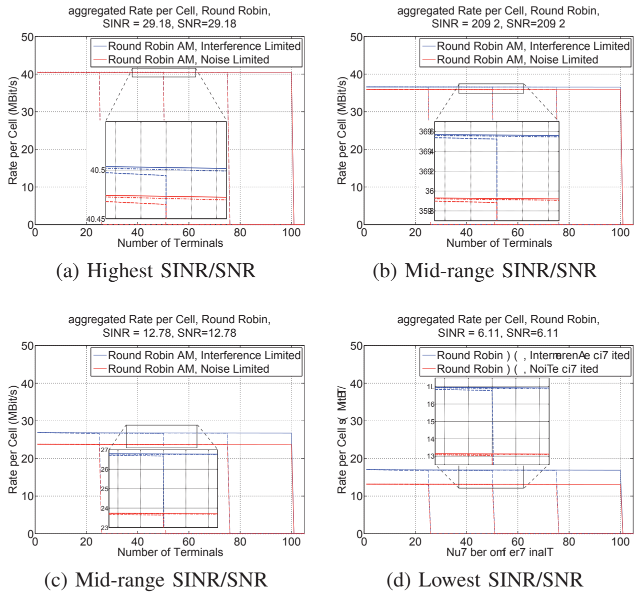
In this section, we compare the aggregated rates per cell in a noise- and an interference-limited system using OFDMA or round robin as scheduling schemes. We use four different SINR values for our calculations, namely 29.18 dB, 20.92 dB, 12.78 dB and 6.11 dB. They correspond to a cell with only a weak interferer (29.18 dB) which evolves to a highly interfered cell (6.11 dB), caused by a growing interferer gain. For each SINR, we choose four different QoS-pairs $\{d_j, \mathcal{P}_j\}$ with a maximum delay d_j and an outage probability $\mathcal{P}_j$ going from very strict constraints of $\{25\text{ ms}, 1\ \%\}$ and $\{50\text{ ms}, 3\ \%\}$, to looser ones with $\{75\text{ ms}, 7\ \%\}$ and $\{100\text{ ms}, 10\ \%\}$. Then, we calculated for a growing number of terminals the resulting aggregated maximum sustainable rates per cell for each of the combinations. This was repeated for the noise-limited case using SNR values of the same level as the SINR values and plotted in Figures 5 and 6. Hence, we can show the different behaviour of rates that interference and noise produce and how big the error becomes by taking false assumptions (i.e. treat an interference-limited system as a noise-limited one). In Figures 5 and 6 blue lines show the results for interference-limited and red lines the ones for noise-limited cells respectively. Different QoS-pairs are distinguished by the line-styles, where the order of break downs fits the order of QoS-pairs from strict to loose.



(a) Highest SINR/SNR (b) Mid-range SINR/SNR

(c) Mid-range SINR/SNR (d) Lowest SINR/SNR

Fig. 5. Aggregated rates per cell for OFDMA and a growing number of terminals. The subfigures show the results for interference-limited systems (blue) and noise-limited systems (red) with several QoS-conditions (linestyles).



(a) Highest SINR/SNR (b) Mid-range SINR/SNR

(c) Mid-range SINR/SNR (d) Lowest SINR/SNR

Fig. 6. Aggregated rates per cell for Round Robin and a growing number of terminals. The subfigures present the results for interference-limited systems (blue) and noise-limited systems (red) with several QoS-conditions (linestyles)

We see that for a high SINR/SNR value, i.e. almost no interference or noise, we obtain approximatively the same results for noise- and interference-limited cases in OFDMA (Figure 5a, and Figure 5b) as well as in round robin systems (Figure 6a and Figure 6b). With decreasing SINR/SNR values, i.e. a growing interferer (or decreasing signal strength for SNR), the differences become much bigger in OFDMA with up to 14 MBit/s, whereas in round robin systems the differences stay at rates of less than 4 MBit/s. Hence, an interference-limited OFDMA system can hardly be compared to a noise-limited system considering possible maximum sustainable rates, whereas round robin approaches would only

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produce small differences. To clarify these differences, they are depicted in Figure 7 and Figure 8. For these figures we lowered the rates for the interference-limited cases by the ones for the noise-limited cases to emphasize the differences. Clearly, for lower SINR values the differences become much bigger in OFDMA systems. The differences considering different QoS

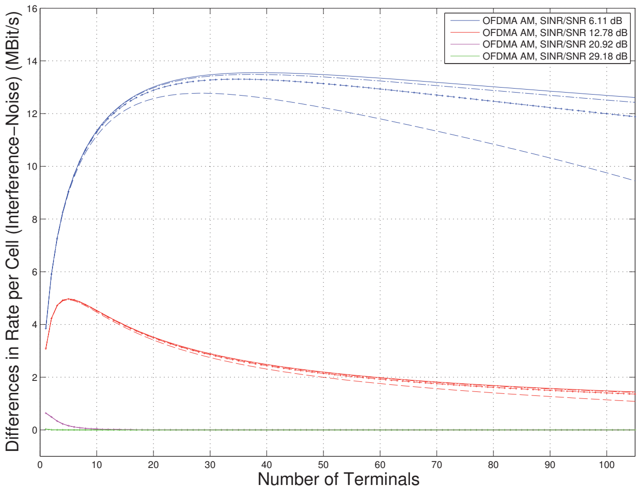

Fig. 7. The figure shows the differences between noise- and interference-limited systems dependent on SNR/SINR and QoS-parameters using OFDMA. Different SINR values differs in colors, while different QoS-settings differs in linestyle.

settings for the round robin approach are smaller than in the OFDMA case with nearly no discrepancies for the round robin schemes.

VI. Conclusions

In this paper, we presented an effective service capacity analysis of interference-limited cellular systems. As shown before, the analysis allows the accurate prediction of the queuing performance of such systems under two contrary

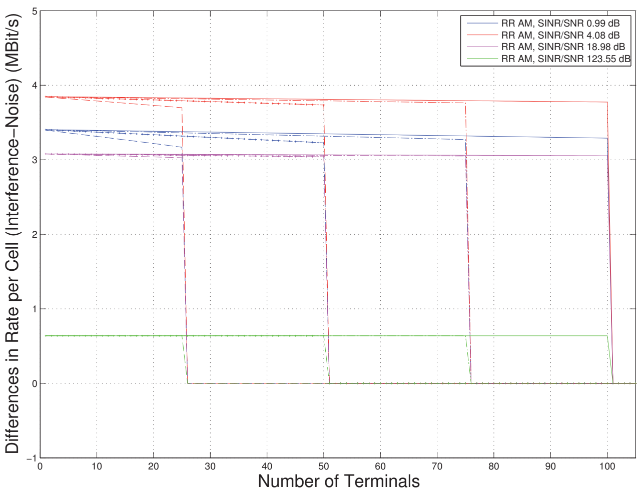

Fig. 8. Displayed are the differences between noise- and interference-limited systems dependent on SNR/SINR and QoS-parameters in a round robin system. Different SINR values differs in colors, while different QoS-settings differs in linestyle.

scheduling policies: a dynamic scheduling scheme (opportunistic OFDMA) as well as static approach (round robin). This is useful for admission control or handover decisions. For example, our analysis can be used to determine the maximum number of real-time multimedia flows that can be supported with sufficient delay and outage probability by the system under given interference constraints from neighboring cells. Based on this number the provider can decide to allow or deny new flows to access the network. Furthermore, our analysis reveals in particular, that interference can not simply be modelled by accounting for it as constant noise. This leads in both scheduling schemes to a significant underestimation of the system. As future work, we plan to investigate the impact of spontaneous interference on such systems as generated by (cognitive) secondary systems with imperfect sensing.

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