Comparison of heuristic and optimal subcarrier assignment algorithms

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Abstract *Wireless multi carrier approaches can support the simultaneous transmissions to several wireless terminals, each one getting only some of the available subcarriers. Judiciously assigning subcarriers to terminals can be used to, e.g., increase the capacity of a wireless cell; however, the resulting assignment problem is difficult. The first subproblem is to decide the number of carriers assigned to each terminal. Then, carriers have to be assigned to individual terminals. For this assignment problem, we present in this paper an optimal but impractical algorithm as well as two new heuristic ones, which have a considerably lower complexity and can be used in real systems. We show that the performance of even the simple heuristic algorithm is comparable with that of the optimal algorithm. The advanced heuristic increases the performance further, at a modest increase in complexity. Both heuristics are at most 5% off from the optimal algorithm's performance*[1].

Keywords: OFDM, dynamic resource assignment, subcarrier assignment, capacity optimization

I. Introduction

Future wireless communication systems will have to meet the requirement of providing high data rate transmissions to wireless terminals. However, simply increasing the symbol rate to fulfill this requirement leads to significant performance degradation through **I**nter**s**ymbol **I**nterference (*ISI*) . To overcome this problem, **O**rthogonal **F**requency **D**ivision **M**ultiplexing (*OFDM*) has been suggested as a possible transmission scheme [1], [2]. In OFDM systems the given bandwidth is divided into many narrowband subchannels: Instead of transmitting a data stream on a single carrier with a symbol rate determined by the desired bit rate, the data stream is divided into many low rate streams, which are transmitted in parallel on the subchannels. Symbol rates on each subchannel decrease, which reduces the impact of ISI, while the overall bit rate of the system stays the same. In addition to the mitigation of ISI, OFDM also has a high spectral efficiency, compared to traditional **F**requency **D**ivision **M**ultiple **A**ccess (*FDMA*) systems. Spacing the carrier frequencies of the subchannels apart by certain constant distances, which are related to the symbol rate of each subchannel, it is possible to obtain more frequency slots without suffering from **I**nter**c**hannel **I**nterference (*ICI*) .

In general, OFDM systems have the following properties:

- Channel gains vary from subcarrier to subcarrier for a single wireless terminal due to multipath propagation,
- Channel gains of each subcarrier vary over time due to the movement of the terminals and other objects within the surrounding area,
- Channel gains of a specific subcarrier vary from wireless terminal to wireless terminal due to statistical independence.

Several adaptive schemes have been investigated which exploit these variations in different ways to improve the transmission of data in OFDM systems.

Considering the varying channel gains of different subcarriers from the viewpoint of a single wireless terminal, schemes known as bit loading algorithms [3], [4] have been suggested which adapt transmission power or bit rates optimally to the channel gains of different subchannels, where either a feasible overall bit rate or a maximum available transmit power is given. They are based on a result from information theory describing how to distribute transmission power over a set of subchannels with different channel gains in order to achieve the channel's capacity. This is known as water filling theorem [5].

In a multiuser transmission scenario, channel gains vary also from terminal to terminal for a given time. Here, the choice of the multi access scheme determines the flexibility of adaption of the schemes. In a **T**ime **D**ivision **M**ultiple **A**ccess (*TDMA*) scheme for example one of the terminals is assigned the complete amount of bandwidth for a certain time interval. In an OFDM system this would result in the assignment of all subcarriers to this ter-

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minal. Due to the variation of the subcarrier channel gains for this terminal, channel gains for some subcarriers might be quite low, while they are quite high for other subcarriers. Due to the channel gain variations regarding one subcarrier for different terminals, the subcarriers with a low channel gain for the one terminal might experience a high channel gain for some other terminal. TDMA, however, does not provide the flexibility to adapt fully to these channel gain variations.

Alternatively using **F**requency **D**ivision **M**ultiple **A**ccess (*FDMA*) , the desired flexibility would be present. But *statically* assigning certain subcarriers to terminals without adapting to the channel gain variations would not exploit the flexibility provided by FDMA. As a consequence, any *fixed* assignment of subcarriers to terminals will waste either power or bit rate.

As an improvement, methods for *adaptively* assigning subcarriers to terminals have been investigated. These adaptive subcarrier assignment algorithms can be geared to decrease the power consumed to transmit at a given achievable bit rate per terminal or to increase the bit rate per terminal while the available power is bounded. However, when computing an optimal assignment for a given description of all subcarrier gains regarding all wireless terminals, complexity is an issue. Recent work [6], [7], [8] has shown a trade off between achieved results regarding the chosen metric and complexity of the assignment algorithm.

In order to minimize the transmitted power at a given bit rate per terminal, an algorithm was derived by Wong et al. [6] which applies Lagrange optimization. Due to the high complexity of this optimization technique, Kivanc et al. presented suboptimal heuristics for the same problem [7]. While these heuristics do not achieve the optimal lower bound on consumed power, they are much cheaper in terms of complexity.

The alternative problem, maximizing the bit rate of multiple wireless terminals while the transmit power is upper bounded, can be decomposed into two tasks: first determining the number of subcarriers each terminal is assigned and then choosing the concrete subcarriers to be assigned. Yin et al. [8] suggest a scheme where first the number of subcarriers assigned to each terminal is determined by considering each terminal's average channel gains and its required bit rate. Then, the assignment of subcarriers to terminals is done.

However, assigning subcarriers to wireless terminals is not a trivial operation, even if the number of subcarriers to be assigned to each terminal is already determined. Yin et al. suggest to solve this problem by using a graph algorithm solving the maximum weight perfect matching problem. While this algorithm computes the optimal assignment of subcarriers to terminals, it is still fairly computationally expensive. Therefore, the question of heuristic approximations to this problem is relevant.

In this paper, we present two related heuristic algorithms for the problem of assigning subcarriers to terminals, once the number of subcarriers assigned to each terminal is fixed – solving the second task in Yin et al.'s scheme. Depending on the computational complexity one is willing to spend, the heuristics achieve a bit rate per terminal almost as high as the optimum solution's rate. The computational complexity of the heuristics is, on the other hand, considerably smaller than that of the optimal solution. Therefore, our heuristics provide a better combination of overhead and resulting performance.

We structure this paper as follows: in Section II we describe the assumed system model. In Section III, we introduce the considered heuristic approaches. In Section IV these algorithms are then compared regarding their performance with assignment schemes providing an upper and lower bound on the performance; their run time values are compared as well. Finally, in Section V we conclude our work.

II. System Model

Assume J wireless terminals are located in a cell with radius R. The wireless terminals move within this cell with a maximum speed $v_{\max}$. We only consider the downlink transmission of data. Time is slotted into units of length T. For data transmission, an OFDM system is employed, providing a total of S subcarriers. At time t, $a_j(t)$ denotes the vector of channel gains of all subcarriers regarding wireless terminal j. Accordingly, $n_j(t)$ is the vector of noise levels of all subcarriers regarding wireless terminal j. The specific channel gain and noise level of subcarrier i regarding wireless terminal j is given by $a_{j,i}$ and $n_{j,i}$, respectively.

$$\begin{aligned} a_j(t) &= (a_{j,1}(t), a_{j,2}(t), \ldots, a_{j,S}(t)) \\ n_j(t) &= (n_{j,1}(t), n_{j,2}(t), \ldots, n_{j,S}(t)) \end{aligned} \quad (1)$$

As a result of the vector definitions of Equation 1, the **C**hannel to **N**oise **R**atio (*CNR*) $g_j(t)$ for terminal j is obtained by Equation 2 [2]

[2] Note that the CNR compares power values. One obtains

$$g_j(t) = \left(\frac{a_{j,1}^2(t)}{n_{j,1}^2(t)}, \frac{a_{j,2}^2(t)}{n_{j,2}^2(t)}, \ldots, \frac{a_{j,S}^2(t)}{n_{j,S}^2(t)} \right) \quad (2)$$

While the channel gains and noise levels change from time unit to time unit, they are assumed to be fixed during one time unit. Essentially, this means that the assumed maximum velocity v_{max} results in an upper bound on T, during which the subcarrier gains do not change significantly. A sufficient condition is to assume the length of one time unit T to be less than the coherence time T_c of the fading process, defined according to [9]. The channel gains vary due to path loss, shadowing and fading. At the beginning of time slot t, the access point knows about the CNRs for all wireless terminals, summarized in matrix $G(t)$:

$$G(t) = (g_1(t), \ldots, g_J(t))^T \quad (3)$$

The CNR value for subcarrier s and terminal j, $g_{j,s}(t)$, can be transformed into an **S**ignal to **N**oise **R**atio (*SNR*) value by multiplying it by the assigned power value $p_s(t)$ for subcarrier s, which yields the SNR value denoted by $x_{j,s}(t) = g_{j,s}(t) \cdot p_s(t)$. Afterwards the resulting SNR value yields an achievable bit rate for this subcarrier by a function $F(\cdot)$, which is given by the considered modulation types and the maximum tolerable symbol- or bit error probability.

At the access point of the cell data streams arrive, one destined for each terminal. As first step the access point then allocates for each wireless terminal the number of subcarriers to be assigned. These subcarrier allocations may change over time, if one considers variable bit rate streams for example. Let the number of allocated subcarriers for the j-th wireless terminal for the next time instance t be given by $z_j(t)$. The goal is now to assign specific subcarriers to each wireless terminal such that the total bit rate is maximized, which will also be called capacity of the cell. Note that we assume enough data to be queued at the access point for each terminal. Denote the assignment of subcarrier s to wireless terminal j during time instance t by $c_{j,s}(t) = 1$; in the other case this value is set to zero. Thus, an optimization problem can be formulated, following [8].

$$\max \sum_{\forall j,s} c_{j,s}(t) \cdot F\left(p_s(t) \cdot g_{j,s}(t)\right) \quad (4)$$

subject to:

$$\forall j : \sum_{\forall s} c_{j,s}(t) = z_j(t)$$

$$\forall s : \sum_{\forall j} c_{j,s}(t) = 1$$

We assume that the resulting assignment can be perfectly (no costs or errors) signaled to the wireless terminals to inform them which modulation types and subcarriers they are supposed to use.

III. Subcarrier Assignment Algorithms

Assigning each wireless terminal the best possible number of $z_j(t)$ subcarriers in terms of CNR is equivalent to the maximum weighted perfect matching problem in bipartite graphs [10]. An optimal solution can be generated by the Hungarian algorithm [11], which has a complexity of $O\left(S^3\right)$, S denoting the number of subcarriers. We present now two related heuristics for this problem. For a detailed description of the algorithms presented here refer to [12], [13].

A. basic Dynamic Algorithm – bDA

As **b**asic **D**ynamic **A**lgorithm (*bDA*) we introduce the following.

```
Initialize(S,G(t));
Iterate over Terminal j
sorted by Priority p
{
 SORT(S,g_j(t));
 ASSIGN_BEST(z_j(t),S);
}
```

At some time t, the algorithm will assign priorities 1 to J to each wireless terminal, where priority 1 is the highest one. The algorithm iterates for each time unit over all terminals, sorted by their current priority. In each iteration step, the considered terminal is assigned the $z_j(t)$ best subcarriers available in terms of CNR values. Afterwards, these subcarriers are removed from the list of available subcarriers. Clearly such an algorithm assigns the wireless terminal with the highest priority subcarriers with a better quality than it assigns the wireless terminal with the lowest priority. To balance this unfairness, the priorities are switched after each time unit. The priority of each wireless terminal is decreased by one for each time unit. The wireless terminal with the highest priority (priority 1) receives for the next time unit the lowest priority (priority J). The initial setting of priorities is generated randomly.

The complexity of this algorithm is given by $O\left(J \cdot S\ log(S)\right)$: The algorithm assigns subcarriers

the power values from the amplitudes by squaring the amplitude values and dividing by the expired time.

to J wireless terminals. The subcarriers are chosen out of a sorted list, which has to be generated for each assignment newly from the yet remaining subcarriers. Sorting a set of n elements requires an algorithmic complexity of $O(n\ log(n))$.

B. advanced Dynamic Algorithm – aDA

As **a**dvanced **D**ynamic **A**lgorithm (*aDA*) we introduce a scheme where additional information — the *weight* — about the subcarriers is used. When considering a particular terminal for subcarrier assignment, the weight of a subcarrier expresses how well this subcarrier might be used by all other terminals with a lower priority than the currently considered one. A weight $w_{j,s}(t)$ of a subcarrier s for wireless terminal j is given by the sum of all CNR values of this subcarrier regarding all wireless terminals with a lower priority than terminal j has.

$$w_{j,s}(t) = \sum_{\forall \text{Ter. t with lower priority than j}} g_{t,s}(t) \quad (5)$$

The aDA extends the bDA by always selecting the subcarriers with the highest possible weight ratio between CNR value and weight. This *weight ratio* of subcarrier s with respect to terminal j is defined as

$$\frac{g_{j,s}(t)}{w_{j,s}(t)}.$$

As in the basic algorithm, subcarriers are assigned to terminals in order of terminal priority. A terminal j is assigned those z_j subcarriers with the largest weight ratio. After an assignment of subcarriers to a terminal, weights for all as yet unassigned subcarriers are recomputed and sorted with respect to the next terminal to be assigned.

This algorithm has the same complexity of $O\left(J \cdot S\ log(S)\right)$ as the bDA has, in terms of the "O" notation. For each assignment step, though, the algorithm has to sort the remaining subcarriers by the weight ratios, which have to be computed first. Therefore, the run time for the aDA is higher than for the bDA. An algorithmic example of the aDA is given below.

```
Initialize(S,G(t));
Iterate over Terminal j
sorted by Priority p
{
 w_j(t)= CALC_WEIGHT(S,j);
 r_j(t)= CALC_RATIO(g_j(t),w_j(t));
 SORT(S,r_j(t));
 ASSIGN_BEST(z_j(t),S);
}
```

IV. Performance Comparison

In this section we present numerical results of the introduced subcarrier assignment algorithms. For comparison reasons we always include also the results of the optimal solution achieved by the Hungarian algorithm. Also, we provide the results when no dynamic assignments of the subcarriers are done, i.e., a fixed set of $z_j(t)$ subcarriers is always assigned to wireless terminal j. This scheme will be called the **s**tatic **S**ubcarrier **A**ssignment (*sSA*) .

A. Simulation Settings

For the simulation, we chose the following settings. The number of available subcarriers is $S = 48$, equivalent to systems such as IEEE 802.11a [14]. The symbol length of one OFDM symbol is chosen to be $4\ \mu$s. We investigate a case where five different modulation types are available: BPSK, QPSK, 16-QAM, 64-QAM and 256-QAM. A modulation type is chosen if it provides the highest bit rate while the symbol error probability is below 10^{-2}.

We vary the number of wireless terminals within the cell from 1 to 48. $z_j(t)$ is chosen such that all wireless terminals receive the same number of subcarriers. Note that the generation of this number is not part of the considered algorithms; instead a resource allocation algorithm delivers this number in advance. Choosing the same number of subcarriers to be assigned to each wireless terminal happens purely for illustration purposes. If one considers a case, where constant bit rate traffic is to be conveyed via the downlink of the cell to all wireless terminals, one might end up with this situation, that the access point allocates the same number of subcarriers for each terminal. The constant number of terminals in the cell is justified by assuming this to be a average over a certain time span, where multiple terminals arrive and departure in the cell.

In order to transmit data from the access point to the wireless terminals, we use the same transmit power per subcarrier. This number varies from -13 dBm to 7 dBm. An IEEE 802.11a compliant system employs in the lower U-NII frequency band (5.15 – 5.25 GHz) a total transmission power of 40 mW, which corresponds to a transmit power of -7 dBm per subcarrier.

The wireless terminals move with a random speed through the cell. The maximum speed is chosen to be 1 meter per second. The considered cell has a radius of 100 meters. Every 2 ms the dynamic algorithms generate new assignments depending on the current CNR matrix describing the system. Once the subcarriers have been assigned, for each assigned subcarrier the highest available modulation type is chosen. In the case of the static assignments, every 2 ms only new modulation types are chosen.

The CNR values of the subcarriers change due to path loss, shadowing and fading. Path loss is determined by the formula $\frac{P_0}{P_t} = K \cdot \frac{1}{d^\alpha}$, where $\frac{P_0}{P_t}$ denotes the ratio between received and transmitted power, d denotes the distance between transmitter and receiver, K denotes the reference loss for the distance unit d is measured in and α is the path loss exponent. We parameterize the reference loss with $10\ \log(K) = 46.7\ \text{dB}$ and the path loss exponent with $\alpha = 2.4$. The shadowing is assumed to be log normal distributed with a standard deviation of $\sigma = 5.8\ \text{dB}$ and a mean of $0\ \text{dB}$ For the time-selective fading, the maximum speed is chosen to be 1 meter per second, while the power spectral density is chosen to have a Jakes-like shape typical for isotropic scattering models [15]. The multipath propagation environment is characterized by a delay spread of $\Delta\sigma = 0.15\mu\text{s}$ with an exponential power delay profile according to the large open space model of ETSI C [16]. An example environment corresponding to such a setting would be a large airport or exposition hall, crowded with people.

The simulation results are based on generated trace files. For each subcarrier of each wireless terminal we generated 150000 correlared CNR samples. For these samples we then considered the performance of the different introduced algorithms.

B. Discussion of Results

As stated in Section II, the objective function of the maximization problem considered here is the overall bit rate of the cell, which is called the capacity of the cell. In Figure 1 and 2 the cell's capacity is shown for a varying number of wireless terminals in the cell. The two figures differ in the amount of transmit power spent per subcarrier, which is in the first case equal to -7 dBm, while in the second case the transmit power is chosen to be 0 dBm. In both figures the capacity behavior is given for all mentioned subcarrier assignment schemes.

In both cases it is evident that assigning subcarriers dynamically is of some benefit. This advantage increases the more wireless terminals are in the cell, due to the increase in diversity in this case. The capacity advantage of the dynamic algorithms varies between 20% and 40% in the case of -7 dBm transmit power; increasing the transmit power to 0 dBm still shows a capacity advantage of up to 30%. A further increase of the transmit power does results in a

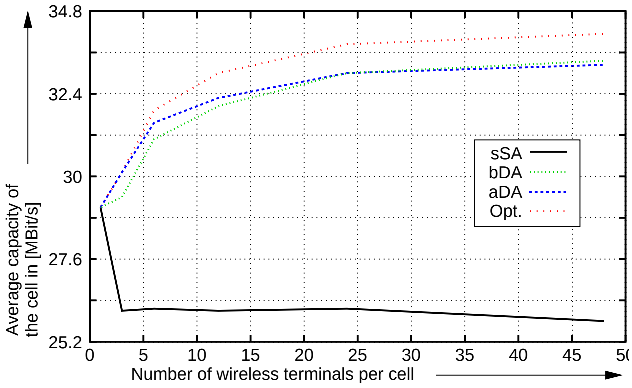

Figure 1. Capacity of the cell for a varying number of wireless terminals in the cell for the sSA, bDA, aDA and the optimal assignment in case of a transmit power per subcarrier of -7 dBm

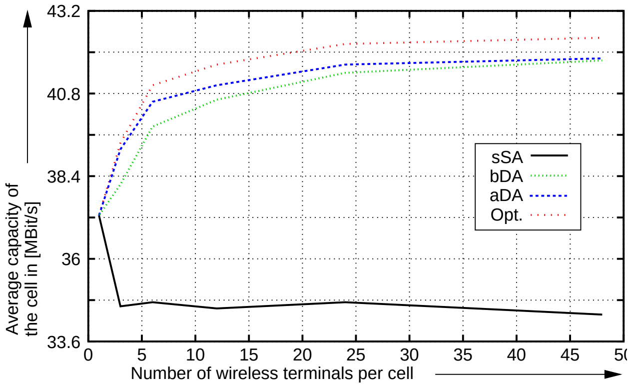

Figure 2. Capacity of the cell for a varying number of wireless terminals in the cell for the sSA, bDA, aDA and the optimal assignment in case of a transmit power per subcarrier of 0 dBm

further reduction of this gap, while a decrease of the transmit power results in a performance difference of at most 50%.

Comparing the aDA to the optimal assignment gives a quite constant capacity advantage of around 3% for the optimal assignments, while the aDA outperforms the bDA by the same percentage in the case of 0 dBm. For a transmit power of -7 dBm, the heuristics are closer together, however, the optimal solution performs a bit better in this case. Varying the number of wireless terminals in the cell does not influence the performance advantage of the optimal assignments; instead it influences the performance gap between the aDA and the bDA such that the aDA performs better if less terminals are in the cell.

Figure 3 shows the average number of bits transmitted per subcarrier for a variable transmit power in the case of 48 wireless terminals. Independent of the transmit power, the dynamic algorithms have a

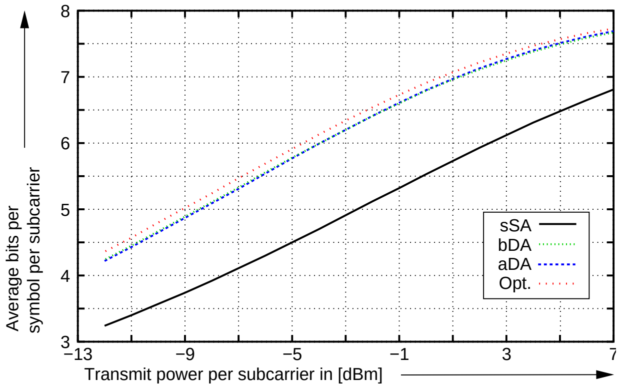

Figure 3. Average number of bits transmitted per subcarrier for the sSA, bDA, aDA and the optimal assignment for a varying transmit power in the case of 48 terminals present in the cell

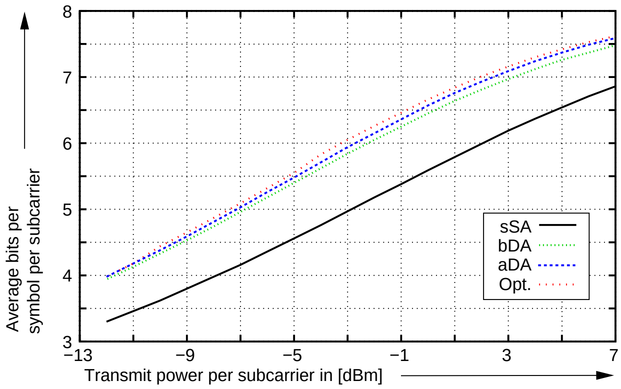

Figure 4. Average number of bits transmitted per subcarrier for the sSA, bDA, aDA and the optimal assignment for a varying transmit power in the case of six terminals present in the cell

$6\ \mathrm{dBm}$ advantage over the static scheme. Within the dynamic algorithms there is no significant difference between the heuristic approaches while the optimal algorithm achieves a slightly better performance, which vanishes for high transmit power levels.

In Figure 4 this average number of transmitted bits is given for the case of only 6 wireless terminals being present in the cell. In this case, the transmit power advantage reduces to $4\ \mathrm{dBm}$ due to the reduced diversity with only six wireless terminals. The dynamic algorithms differ slightly, such that the aDA achieves a better result than the bDA. Again the optimal algorithm performs best, however the performance gain compared to the aDA is only marginal.

At last, we present numerical values illustrating the run-time behavior of the different subcarrier assignment algorithms. For a cell with $S = 48$ subcarriers and a varying number of wireless terminals we measured the average run-times of the three different dynamic algorithms. The computer used for the measurements consisted of an Intel Pentium 4 CPU with $1.8\ \mathrm{GHz}$ and 512 MBytes of memory. In order to generate the optimal assignment, we used the LEDA library version 4.1 (http://www.mpi-sb.mpg.de/LEDA/). The results are given in Figure 5.

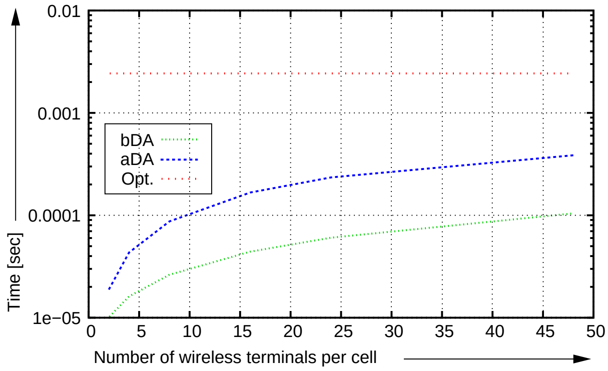

Figure 5. Logarithmic plot of average run time values for the bDA, aDA and the optimal assignment with a fixed number of subcarriers ($S = 48$) and an increasing number of wireless terminals in the cell

While the run times for the heuristics increase with an increase of the number of terminals in the cell, the optimal algorithm has a constant run time. This is due to the working principle of the optimal algorithm: it has to span a bipartite graph, where the left nodes corrspond to the terminals and the right ones to the subcarriers. Even if less terminals are present, the bipartite graph has to be expanded to a one with the same number of nodes on the left and right side (this is done by copying all node connections of one terminal, if less terminals are present; it inidcates then that this terminal will receive more than one subcarrier by the matching algorithm). Therefore, the algorithm has always to find the same amount of matchings in a bipartite graph with 48 nodes to the left and right in this case. Hence the run times are constant, even if only a few terminals are presented. In contrast, the the heuristics perform faster if less terminals are present. The gap between bDA and aDA increases, beginning at $10\ \mu\mathrm{s}$ for 2 terminals present and ending at $300\ \mu\mathrm{s}$ for 48 terminals present. While the optimal algorithm consumes $2.5\ \mathrm{ms}$ in order to generate new assignments, the bDA needs a maximum of $100\ \mu\mathrm{s}$ for generation and the aDA consumes about $400\ \mu\mathrm{s}$. Note that we assumed here that every $2\ \mathrm{ms}$ new assignments are generated by the access point. In reality, the optimal algorithm would not be able to serve as assignment algorithm in such a case ! Also, due to the much higher run time of the optimal algorithm, it requires subcarrier gains to be stable much longer and can therefore only be used up to a much lower maximum speed of wireless terminals. The heuristics instead can deal with much faster changing wireless channels.

V. Conclusions

In this paper we proposed two related algorithms solving the subcarrier assignment problem in OFDM systems. While optimal solutions exist, our algorithms are heuristic in nature. All dynamic algorithms differ in terms of computational complexity, where a "basic" version of the heuristic approach is computationally least expensive, followed by an "advanced" version and the optimal solution.

The introduced schemes are compared in terms of average throughput and cellular capacity in a realistic environment. In addition, a static assignment scheme serves as as lower bound. The results are obtained via simulation. Also, the dynamic schemes are compared in terms of average run time.

The comparison of these schemes shows that in general the usage of a dynamic scheme pays off well. On average, the dynamic schemes outperform the static scheme by 30 – 40% throughput. Moreover, the difference between the dynamic schemes is at most 5% of throughput between the basic heuristic approach and the optimal solution. The advanced heuristic approach is between these two schemes and performs better for a smaller number of terminals in the cell compared to the basic heuristic approach. While the computational complexity expressed by the "O" notation already indicates a difference between the heuristic algorithms and the optimal one, run time measurements demonstrate that the heuristics are up to twentyfive times faster than generating the optimal assignment. Taking this into account, the heuristics provide a very good throughput behavior compared to the optimal solution, where the advanced version is especially attractive in cases with a low number of terminals in the cell, due to its low run time in these cases and its higher performance. The higher the number of terminals gets, the more does the basic algorithm become attractive, since its run time is four times lower for a high number of terminals in the cell while the performance approaches that of the advanced version.

As further work we consider the following aspects. We have made strong assumptions regarding the signaling system and the subcarrier state information. The question arises by how much performance improvements are affected if the signaling effort has to be taken into account instead of assuming a perfect and cost free signaling system. Also, the performance will probably be decreased by realistic subcarrier state information which might be to some extent erroneous, depending on the speed of the termnals for example.

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