Reducing Outage Probability Over Wireless Channels Under Hard Real-time Constraints

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***Abstract*—Data transmissions over wireless fading channels face a fluctuating environment impacting the transmission quality. Hard real-time scenarios typically require a very reliable transmission in terms of a successful reception of a message within quite short deadlines. Implementing wireless systems for such kind of quality-of-service requirements necessitates taking actions to prevent or – at least – alleviate the shortcomings of wireless transmissions. This work deals with theoretical implications of the introduction of frequency/spatial domain diversity on the outage probability of messages with a given size and deadline. It is shown that using a capacity overload-redundancy strategy relying on a multitude of independent channels can significantly improve the probability of successful reception in a hard real-time context.**

I. Introduction

Real-time sensitive message exchange comprises two fundamental aspects: The message has to be received correctly and timely by the intended recipient. Up to date, hard real-time message exchange is usually realized by wired connections, especially if the associated *outage probability* (the probability with which the transmitted message is not received correctly within the required time span) is very low. For instance, the required outage probabilities in real-time control of industrial automation are typically in the range of 10^{-6} to 10^{-10}, while the message size typically spans a few bit up to 100 byte. These messages need to be conveyed within a few milliseconds down to 100 microseconds as most stringent real-time requirement. Fiber-optical cables are employed for such application data to ensure the ultra-high reliability constraint. On the other hand, using cables not only reduces the flexibility but also causes installation and maintenance costs. Hence, there is research interest in employing wireless transmission systems instead of using cables [1], [2].

However, it is entirely open if wireless systems can achieve this quality-of-service (QoS) level at all and if so, how to design such a wireless system. Obviously, the most challenging obstacle to overcome is the stochastic behavior of the wireless channel. It is well known that the quality of the wireless channel might fluctuate heavily on rather short timescales. This is especially true for industrial environments [3], where a variety of noise sources and many metallic objects can be found in the vicinity of a possible radio channel, giving rise to the expectation of fatal radio channel conditions.

Developments of wireless transmission systems over the last decades have shown an immense progress. Theoretical boundaries for different scenarios [4], [5] were derived. Most of this work bases upon idealized assumptions and does not consider the tight delay requirements of hard real-time transmissions in their analyses. Furthermore, many papers, e. g., [6], lack the design of practical strategies which base upon results derived in those papers. Caire et al. [7] have shown that for a basic repetition diversity scheme, there is an optimal repetition order. But this scheme relies on transmitter channel state information and optimal power allocation, and the optimum can only be determined numerically. On the other hand, however, many approaches and algorithms [2], [8] are proposed that are dealing with real-time aspects. To the best of the authors' knowledge, the performance gains of those approaches are often not stated precisely in terms of general error probability for a given system, or requirements of industrial automation are not considered.

Being able to mitigate impacts of radio channel fluctuations on the transmission quality does not necessarily mean that the employed strategies are also able to cope with fluctuations in a real-time message exchange. This is because its real-time characteristics push quite a few achievements in the evolution of wireless systems aside. Due to very low delay requirements, sophisticated error correction codes need to be ruled out unless the cost of the equipment is of less importance. Furthermore, channel-state adaptive transmission strategies cannot be adopted as there is simply no time to acquire the current channel state. Also retransmission strategies can not be applied due to the delay requirements. However, the application of diversity techniques is well known to improve the reliability of data transmission over wireless channels in general [9].

In this paper, we follow this direction with a more specific focus. Wireless communication systems have undergone a rapid development towards truly broadband systems and can be expected to evolve further in this direction. Key technologies that enable this development are orthogonal frequency division multiplexing (OFDM) as well as multiple input multiple output (MIMO) transmission systems. While wireless modems employ more and more bandwidth, the message size in real-time applications typically stays constant and is very small in comparison to typical IP packet sizes. Hence, even though stringent deadlines have to be met, the message sizes are so small that in OFDM and/or MIMO systems the message might be transmitted many times over many different *independent* channels. Given a set of independent channels as well as a certain message size, the question arises how to split up the message over the channels, i. e., how much redundancy to employ and how to distribute it over the channels. The contribution of this paper is the investigation of these different options mathematically as well as numerically. We observe that for any combination of average channel gain, delay requirement and message size, there exists an optimal degree of redundancy together with a transmit power level such that the outage probability is extremely minimized. Surprisingly, the amount of redundancy leading to this "sweet spot" is neither simply retransmitting the message n times if n is the number of available channels, nor is the optimal strategy to split the message in n fractions and then transmit each fraction over a separate channel. Instead, the best way turns out to be a setting in between these two extremes. Although this proposed transmission system is conceptually simple, it is very effective by providing extremely low outage probabilities.

Following this introduction, we present the chosen system model and the derivation of basic outage probabilities in Sec. II. After that, the implications of exploiting channel diversity by different redundancy settings for a real-time transmission over a wireless channel are investigated in Sec. III. Sec. IV contains some numerical evaluation. Finally, conclusions of this work are given in Sec. V.

II. System Model

We consider a hard real-time transmission between one transmitter (TX) and one receiver (RX). Messages of size I_{out} need to be transmitted successfully within a maximal tolerable delay D_{out} sec. If the message is not received correctly within this time span, the system faces an erroneous situation referred to as *outage*. We seek a design that minimizes the outage probability. Furthermore, we focus on small messages in the order of several byte and tight delay constraints being in the range of a couple of milliseconds. To simplify the discussion, we neglect propagation delay and processing delay at the transceivers (assuming both effects to be in the range of microseconds) such that the delay D_{out} can be spent entirely on the transmission of the message.

A. Transmission System and Wireless Channel

An OFDM-based software-defined radio serves as the wireless transmission system. This system covers a total bandwidth of W Hz being subdivided into K sub-channels. Each sub-channel experiences frequency flat fading, i. e., sub-channel bandwidths are smaller than the channel coherence bandwidth. Out of K potential sub-channels, L statistically independent fading sub-channels are chosen for data transmission. They are selected such that L is maximized. These L sub-channels span a total bandwidth of B. Each sub-channel is employed with the same transmit power denoted by P_{TX}. Accordingly, total transmit power is specified by $P_{\text{TX,tot}} = L \cdot P_{\mathrm{TX}}$. This available total transmit power $P_{\text{TX,tot}}$ is fixed, i. e., the more sub-channels are picked, the less power is available per sub-channel. Message transmission spans exactly one time frame of length D_{out}.

Channel gain per sub-channel is only composed of path loss and Rayleigh fading. More precisely, we model the wireless sub-channels as (independently) flat, block-fading Rayleigh-channels, i. e., the fading impact is constant for the whole considered time frame and sub-channel. The RX has perfect knowledge of the channel, whereas no channel state information is available at the TX. Shadowing, mobility, and possible other interference sources are not considered. Path loss is assumed to be constant in our investigations. Thus, the average received power for one sub-channel is given by:

$$P_{\mathrm{RX}} = \frac{P_{\mathrm{TX}}}{d^{\alpha}} = \frac{P_{\text{TX,tot}}}{L \cdot d^{\alpha}}, \quad (1)$$

in which d is a fixed distance between TX and RX, and α the path-loss exponent.

Due to Rayleigh-fading, the instantaneous received power changes randomly from frame to frame. We denote the stochastic variable of the received power per frame by $\mathcal{P}_{\mathrm{RX}}$. Following this model, $\mathcal{P}_{\mathrm{RX}}$ is exponentially distributed such that the probability density function (PDF) of the received power is given by:

$$\rho_{\mathcal{P}_{\mathrm{RX}}}(x) = \frac{1}{P_{\mathrm{RX}}} \cdot \exp\left[-\frac{x}{P_{\mathrm{RX}}}\right] \quad (2)$$

P_{RX} represents the average received power. Noise power P_{N} is set to be $P_{\mathrm{N}} = N_0 \cdot \beta$, where $N_0 = 4.142 \cdot 10^{-21}\, {}^{W}\!/_{\mathrm{Hz}}$ denotes the spectral noise power density, and β the respective bandwidth. Hence, the instantaneous signal-to-noise ratio (SNR) is given by $\gamma = \frac{\mathcal{P}_{\mathrm{RX}}}{P_{\mathrm{N}}}$ with the corresponding average SNR $\overline{\gamma} = \mathbb{E}\left[\gamma\right] = \frac{P_{\mathrm{RX}}}{P_{\mathrm{N}}}$.

B. Power-rate Function and Outage Capacity

We base our investigation upon the well-known Shannon capacity formula [10]:

$$C = \beta \cdot \log_2\left[1 + \frac{P_{\mathrm{RX}}}{P_{\mathrm{N}}}\right], \quad (3)$$

which relates the considered bandwidth β and the SNR to the achievable capacity C. An upper bound on achievable capacity (information rate) is represented by Eq. (3) only if special conditions are met, e. g., certain statistical assumptions on the input signal and the noise figure are fulfilled. An important condition is that the employed code book has to provide code words which are long enough to cope with the fading process, i. e., compensating fading dips in the time domain such that ergodicity is retained.

In our hard real-time scenario in which we have stringent delay constraints in the presence of a noise-limited Rayleigh-fading channel, that ergodic assumption might not necessarily be met at all times. This means that with a non-zero probability there are cases for which no code word exists which is able to cope with the present channel state because the channel variability within the allowed time frame is too small. As a result, channel capacity C can be regarded to be a random variable depending on another random variable, the instantaneous SNR. The quantiles of this random variable are referred to as *outage capacity* [4].

C. Outage Probability

Hence, *outage capacity* is closely related to the term *outage probability*. Due to the varying nature of the wireless fading channel on the one hand, and stringent timing constraints for a fixed message size in combination with a finite code book on the other hand, there arise situations for which the channel capacity is not large enough to support the transmission of a message. This event is termed an *outage*. Based on the channel fading characteristics, there is a probability associated to that outage event. This probability is called *outage probability*. In this context, the question arises of how to determine the probability that a certain channel capacity can be obtained in case of stochastically distributed received power levels. We first give some basic analysis for the single channel case.

1) Generic Capacity PDF: The first step is to derive a PDF describing the channel capacity by using PDF transformation [11]. We use Shannon's capacity formula stated in Eq. (3) in which P_{RX} is not a fixed power level but given by an arbitrary PDF, $\rho_{\mathcal{P}_{\mathrm{RX}}}$. This yields the generic channel capacity PDF:

$$\rho_{\mathrm{C}}\left(C\right) = \rho_{\mathcal{P}_{\mathrm{RX}}}\left(P_{\mathrm{N}} \cdot \left(2^{\frac{C}{\beta}} - 1\right)\right) \cdot \frac{\ln(2) P_{\mathrm{N}}}{\beta} \cdot 2^{\frac{C}{\beta}} \qquad (4)$$

Applying Eq. (2) to Eq. (4) leads to the capacity PDF of the considered Rayleigh-fading channel:

$$\rho_{\mathrm{C}}\left(C\right) = \exp\left[-\frac{P_{\mathrm{N}}}{P_{\mathrm{RX}}} \cdot \left(2^{\frac{C}{\beta}} - 1\right)\right] \cdot \frac{\ln(2) P_{\mathrm{N}}}{\beta \cdot P_{\mathrm{RX}}} \cdot 2^{\frac{C}{\beta}} \qquad (5)$$

in which P_{RX} is given by Eq. (1).

2) Outage Probability Equation: In the second step, to derive the outage probability ($\mathrm{Pr}_{\mathrm{out}}$) per message, we define outage capacity (C_{out}) as the ratio of message size and its associated delay constraint:

$$C = I_{\mathrm{out}} \;/\; D_{\mathrm{out}}$$

An outage occurs if the channel capacity for that time frame is smaller than the required outage capacity, C_{out}:

$$\begin{aligned}\mathrm{Pr}_{\mathrm{out}} &= \Pr\left\{C < C_{\mathrm{out}}\right\} = \int_0^{C_{\mathrm{out}}} \rho_C(C)\;\mathrm{d}C \\ &= 1 - \exp\left[-\frac{P_{\mathrm{N}}}{P_{\mathrm{RX}}}\left(2^{\frac{C_{\mathrm{out}}}{\beta}} - 1\right)\right]. \qquad (6)\end{aligned}$$

III. Channel Diversity

The introduction of diversity to wireless transmission systems is well known to improve reliability. We follow this approach and seek to reduce the outage probability by employing multiple uncorrelated sub-channels at the same time. In principle, we could also employ multiple independent spatial streams. A key aspect of the separation is that there must not be correlation between the single resulting transmission paths (otherwise, the results derived below do not hold anymore, this is considered for future work). First, we derive the basic concept with respect to outage capacity and corresponding outage probability. In the last part of this section, we comment on some practical issues of our proposed scheme.

We investigate three different approaches which are trading total available channel capacity off against outage probability:

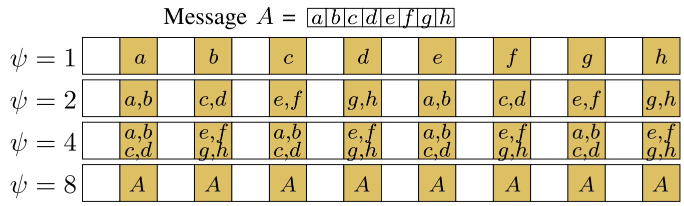

Figure 1. Spreading schemes of outage capacity ($K = 16$, $L = 8$)

1) Splitting up the message onto L adjacent sub-channels, i. e., requiring a down-scaled C_{out} per sub-channel.
2) Repeating the message on each of the L parallel sub-channels, i. e., requiring the full C_{out} per sub-channel.
3) A combining approach in between.

The meaning of 1) corresponds to the case in which the target outage capacity is distributed equally across all available sub-channels, whereas 2) equals the case in which the same target capacity is requested from any of the single channels. In the second case, it is sufficient that at least one channel provides the required capacity at any time. The third attempt uses a mixture of splitting up the information onto several sub-channels with scaled capacity demands, and simultaneously sending each fraction of the original message multiple times.

In order to clarify this, Fig. 1 illustrates the three schemes exemplary for a system which consists of $K = 16$ sub-channels out of which $L = 8$ are sufficiently far apart in the frequency domain such that they are mutually independent. The parameter ψ is called *channel weight factor*. It represents the multiplicative factor of additionally requested capacity compared to the case in which only the target outage capacity is demanded from the set of L channels. For convenience, we allow only those values of ψ (ranging between 1 and L) whose ratio L/ψ gives integer results.

In Fig. 1, message "A" is split into eight fragments of the same size. The first row of Fig. 1 depicts the above mentioned Case 1) in which equal fractions of message "A" are distributed over all available (colored) sub-channels. The resulting desired capacity of every single sub-channel is $\frac{C_{\mathrm{out}}}{8}$. An outage event occurs whenever at least one single fraction on some sub-channel is in outage. In contrast, choosing $\psi = 8$ refers to the above mentioned Case 2). The full outage capacity C_{out} is requested from every single channel as the message is simply repeated L-times. An outage occurs only if all L sub-channels face an outage simultaneously. However in this case, sub-channels might simply be overcharged: A certain bandwidth can only offer capacity up to a certain bound beyond which the probability of a successful transmission converges rapidly towards zero. As compensation it suffices that only one of the L sub-channels offers good transmission conditions.

Fig. 1 further depicts Case 3) for $\psi = 2$ and $\psi = 4$, respectively. Setting ψ to 2, for example, results in an overall outage capacity of $2 \cdot C_{\mathrm{out}}$. Since twice the capacity is demanded from every single channel, four channels are enough to carry the whole message I_{out} within the timespan D_{out}. The remaining four channels can be used to increase redundancy. Every aggregated message fragment is sent twice, for example "(a,b)". Now an outage event occurs only if there is a simultaneous outage event in both copies of the same message fragment. Notice that no partial message retrieval is possible, i. e., if a sub-channel faces an outage, then all parts of the message conveyed in that sub-channel in that particular time frame are lost.

A. Outage Probability for Extreme Cases

For Case 1) and Case 2), the joint outage probabilities are derived in the following two subsections.

1) Joint Outage Probability with Down-Scaled Capacity: Every single (independent) sub-channel out of the set of L available sub-channels has its own probability Pr_{out} to transmit its fraction of desired capacity C_{out}. The transmission will face an outage as soon as only a single sub-channel fails. The resulting joint outage probability $J_{L,\psi}$ can be computed based on Eq. (6) with

$$\beta = \frac{B}{L} \qquad \text{and} \qquad C_{\text{out}} = \frac{I_{\text{out}}}{L \cdot D_{\text{out}}}$$

as follows:

$$\begin{aligned} J_{L,\psi} &= \mathrm{Pr}\left\{\text{outage in at least one of the } L \text{ channels}\right\} \\ &= 1 - \mathrm{Pr}\left\{\text{success in all } L \text{ channels}\right\} \\ &= 1 - \left(1 - \mathrm{Pr}_{\text{out}}\right)^L \\ &= 1 - \left(\exp\left[-\frac{N_0\, B\, d^\alpha}{P_{\text{TX,tot}}}\left(2^{\frac{I_{\text{out}}}{B \cdot D_{\text{out}}}} - 1\right)\right]\right)^L \end{aligned} \tag{7}$$

2) Joint Outage Probability without Down-Scaled Capacity: In contrast to the case with the scaled capacity, every single sub-channel now has to transmit the full capacity C_{out}. An outage occurs only in case all sub-channels face an outage event simultaneously. The resulting joint outage probability $J_{L,\psi}$ can be computed based on Eq. (6) with

$$\beta = \frac{B}{L} \qquad \text{and} \qquad C_{\text{out}} = \frac{I_{\text{out}}}{D_{\text{out}}}$$

as follows:

$$\begin{aligned} J_{L,\psi} &= \mathrm{Pr}\left\{\text{outage in all } L \text{ channels simultaneously}\right\} \\ &= \left(\mathrm{Pr}_{\text{out}}\right)^L \\ &= \left(1 - \exp\left[-\frac{N_0\, B\, d^\alpha}{P_{\text{TX,tot}}}\left(2^{\frac{L \cdot I_{\text{out}}}{B \cdot D_{\text{out}}}} - 1\right)\right]\right)^L \end{aligned} \tag{8}$$

B. Outage Probability for Combining Cases

A message is split up into smaller chunks of information. These message parts can be transmitted multiple times in parallel, whereas every single part requires a certain channel capacity to be transmitted successfully. It is necessary that at least one of the sub-channels over which duplicates of the same message part are transmitted offers the required capacity, but this has to be the case for all single message parts.

The general $J_{L,\psi}$ formula for an arbitrary number of duplicates, ψ, is derived as follows:

$$\begin{aligned} J_{L,\psi} &= \mathrm{Pr}\{\text{outage in at least 1 of the } L/\psi \text{ message parts}\} \\ &= 1 - \mathrm{Pr}\{\text{success (in at least one of the } \psi \text{ duplicates)} \\ &\qquad \text{for all of the } L/\psi \text{ message parts}\} \\ &= 1 - \left(\mathrm{Pr}\{\text{success in at least one of the } \psi \text{ dup.}\}\right)^{\frac{L}{\psi}} \\ &= 1 - \left(1 - \mathrm{Pr}\{\text{outage in } \psi \text{ dup. simultaneously}\}\right)^{\frac{L}{\psi}} \\ &= 1 - \left(1 - \left(\mathrm{Pr}_{\text{out}}\right)^\psi\right)^{\frac{L}{\psi}} \end{aligned}$$

with

$$\beta = \frac{B}{L} \qquad \text{and} \qquad C_{\text{out}} = \frac{\psi}{L} \cdot \frac{I_{\text{out}}}{D_{\text{out}}}$$

leading to:

$$J_{L,\psi} = 1 - \left(1 - \left(1 - \exp\left[-\frac{N_0\, B\, d^\alpha}{P_{\text{TX,tot}}}\left(2^{\frac{\psi\, I_{\text{out}}}{B\, D_{\text{out}}}} - 1\right)\right]\right)^\psi\right)^{\frac{L}{\psi}} \tag{9}$$

Setting ψ to be 1 or L, respectively, leads to the previously in Sec. III-A derived equations for the extreme cases.

C. Practical Issues

We briefly discuss some practical issues that arise in the context of our proposed scheme. In order to derive the optimal channel weight factor ψ, the transmitter has to possess information about the average received SNR, $\bar{\gamma}$, and about the channel spacing for which channels can be assumed to face independent fading characteristics. It has to be noted, though, that there is no need to feed back the instantaneous channel conditions. The only requirement once the general channel conditions change is the need to keep the transmitter up to date regarding the average received SNR, either by means of explicit feedback, or by estimations based on the reverse channel.

Unfortunately, Eq. (9) cannot be solved analytically with respect to ψ. Since the optimal ψ can be found for rather small values, an efficient scheme would be to start with $\psi = 1$ and increase ψ until a raise in joint outage probability is observed. If memory is not a limiting factor, a lookup table with precomputed values can also be considered. It contains optimal ψ values for varying average SNR and different numbers of available channels.

IV. Numerical Evaluation of the Outage Probability

In this section, we evaluate our analysis regarding the joint outage probability, $J_{L,\psi}$, for different settings of message size and transmission system parameters.

A. Analysis Parameters

Transmit power level and path-loss / distance are chosen to realize an average received SNR of $20\,\mathrm{dB}$ at the RX (unless stated otherwise). The OFDM system covers a maximum bandwidth of $W = 3.2\,\mathrm{MHz}$ which is divided into $K = 512$ sub-channels (e. g., by using a 512-FFT) out of which $L = 32$ sub-channels are used for the actual data transmission. This results in a sub-channel width of $6.25\,\mathrm{kHz}$, and, thus, into a total used bandwidth of $B = 200\,\mathrm{kHz}$.

Table I
ANALYSIS PARAMETERS

Abbrev.	Explanation	Value
W	Total available bandwidth	3.2 MHz
K	Number of available sub-ch.	512
B	Total used bandwidth	200 kHz
L	Number of used sub-ch.	32
$P_{\text{TX,tot}}$	Total Transmission power	$0.1\,\text{W} \cdot B\,/\,20\,\text{MHz}$
	Radio channel	$d = 500\,\text{m},\ \alpha = 3.75$
	Delay spread (rms)	$\approx 1.6\,\mu\text{s}$
I_{out}	Message size	$\{64, 128, 256, 512\}$ bit
D_{out}	Message deadline	1 ms
ψ	Channel weight factors	$\{1, 2, 4, 8, 16, 32\}$

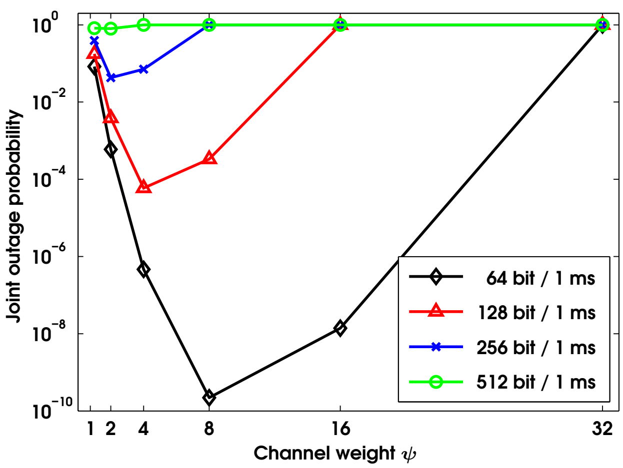

Figure 2. Dependence of joint outage probability on channel weight factor

All important analysis parameters are summarized in Table I together with their respective values. After presenting the basic results, some of these parameters are changed to show the impact their impacts on the outage probability.

B. Joint Outage Probability

As can be seen in Fig. 2, there is a trade-off between channel capacity and outage probability in a real-time sensitive transmission over a Rayleigh-fading channel. It shows the achievable joint outage probability, $J_{L,\psi}$, plotted against the channel weight factor, ψ. The leftmost case ($\psi = 1$) equals the case in which every single sub-channel has to deliver $^1/_{32}$ of the target capacity given by $I_{\text{out}}/D_{\text{out}}$. The rightmost depicted coordinate belongs to the situation in which the full target capacity is requested from all sub-channels. Four different capacities are shown yielding spectral efficiencies ranging from approximately 0.3 to $2.5\,^{bit/s}/_{\text{Hz}}$. As it becomes apparent in Fig. 2, the best strategy is to use a combination of repetition and spreading. Between these two extremes, there is a region in which the joint outage probability can be reduced by several orders of magnitude.

We are aware of the fact that the depicted outage probabilities are not all sufficient for ultra-reliable communication. The intention is to give a first insight into the general behavior. In the following section, we investigate further steps to decrease outage probability in this framework.

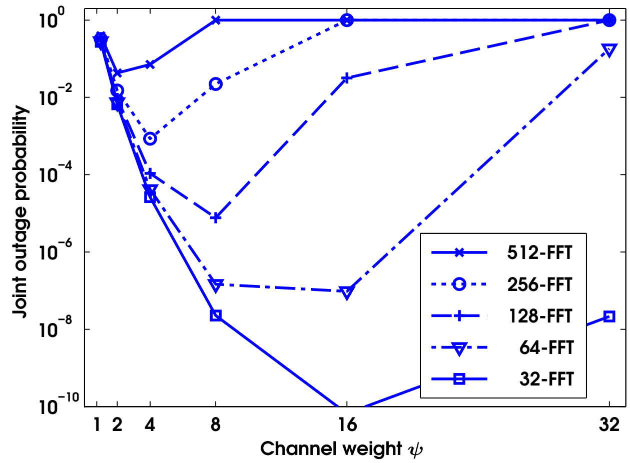

Figure 3. Additional impact of bandwidth (FFT size) on joint outage probability ($\overline{\gamma} = 20\,\text{dB}$, $I_{\text{out}} = 256\,\text{bit}$)

C. Varying Transmission Parameters

Not only the message size and the channel weight factor influence the joint outage probability of our proposed scheme, but also do other parameters, such as available bandwidth and average received power (received SNR). This is of particular importance if the resulting outage probability has to be further improved.

Granted that we face certain channel conditions for which adjacent sub-channels are highly correlated, and that the used device can detect that. If the device is capable of modifying its FFT size while operating, then it might be advisable to increase the per-sub-channel bandwidth on-the-fly by decreasing the FFT size[1]. Note that this increases the total used bandwidth B and, thereby, decreases the average SNR (total transmit power fixed for fair comparison). In Fig. 3, the resulting joint outage probability for a transmission of $I_{\text{out}} = 256\,\text{bit}$ is presented. The solid curve corresponds to the blue curve previously (c.f. Fig. 2). There, the choice of 32 sub-channels and an FFT size of 512 dictate a used bandwidth of $200\,\text{kHz}$ and that only every 16$^{\text{th}}$ sub-channel is used. Reducing the FFT size down to a 64-FFT leads to the situation in which the 32 sub-channels cover half the available spectrum of $3.2\,\text{MHz}$, but also have a remarkably lower average SNR. However, a huge outage probability reduction can be realized due to the linear instead of a logarithmic dependency (c.f. Eq. (3)).

Observing other channel conditions, e. g., twice the delay spread compared to Table I, facilitates another way of extending the bandwidth. This larger delay spread yields a smaller coherence bandwidth, and, thus, higher variability among the instantaneous channel gains. Hence, a larger number of (independent) sub-channels can be found. Once again, each of these sub-channels has the size given in IV-A. The larger

[1]If an OFDM system spans a certain fixed bandwidth, modifying the FFT size results in a varied per-sub-carrier bandwidth.

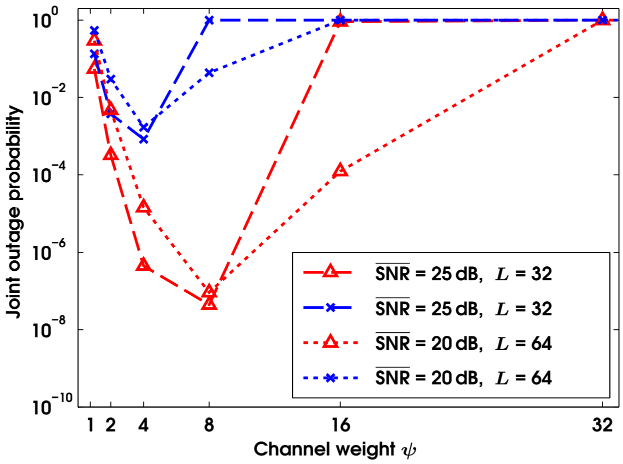

Figure 4. Additional impact of transmit power and bandwidth (number of usable sub-channels) for $I_{\text{out}} = 128\,\text{bit}$ (red) and $I_{\text{out}} = 256\,\text{bit}$ (blue)

number of "usable" sub-channels leads to an increased usable bandwidth. Referring to the dotted curves in Fig. 4, it can be seen that choosing more independent sub-channels can also improve the joint outage probability. Comparing bandwidth extension by means of decreased FFT size and by means of a larger number of small independently fading sub-channels provides another insight: For few sub-channels it is better to increase the number of sub-channels instead of their width. But for a system with a fairly large L, increasing the per-sub-channel bandwidth is advantageous. In the provided figures, the blue dotted curve (with circles) in Fig. 3 represents the same covered bandwidth as the blue dotted curve (with crosses) in Fig. 4, but the curve in Fig. 3 has slightly better performance. The performance difference is continuing to grow when considering even wider bandwidths. In contrast, having less bandwidth is advisable to pick a large number of narrow sub-channels and, thereby, increasing diversity.

Another straight forward approach to boost the system performance is to increase the average SNR $\overline{\gamma}$, i.e., the transmit power. As can be seen in Fig. 4, increasing the transmit power by 5 dB improves the joint error probability remarkably. In that figure, for example, an increase by 5 dB achieves almost the same performance as in the case of twice the number of independent sub-channels.

Note that the considered bandwidth B is comparably small. The joint outage probability can be further decreased to a significant extent by covering a larger available bandwidth W, either by a software-defined radio (SDR) spanning a larger bandwidth, or – in the simplest case – by a parallel approach in which the scheme proposed in this paper is executed multiple times in parallel.

V. Conclusions

Hard real-time communication over wireless channels is a very challenging task since the highly variable wireless channel conditions pose a threat for the strict timing constraints. Employing diversity strategies in different domains is a way to cope with fluctuations of the wireless channel. Typically, retransmissions are a widely used method in wireless networks to increase reliability, but with stringent delay requirements, focus has to be put on other strategies, either in frequency or in the spatial domain.

In this paper, we have investigated overload-redundancy strategies that perform a preventative retransmission by simply conveying the message several times over independent wireless channels with a varying degree of overload. We have shown that for a given message size, deadline and average channel state there exist overload schemes that drastically reduce the outage probability by several orders of magnitude. If any overload scheme overcharges the set of independent channels, a software-defined radio might reach the reduced outage probability by either increasing the transmit power or extending the bandwidth (if possible due to intersymbol interference). As future work, we are currently applying this overload-redundancy scheme in more practical system models that base upon packet-error probabilities. Furthermore, we intend to implement the scheme on a prototyping platform. Further questions relate to a combination with power loading schemes (requires actual channel information on transmitter side), and to the impact of interference on the overload design, both from a theoretical and practical perspective.

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